Robustness Analysis

Eleftherios Siskos

lsiskos@epu.ntua.gr

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(...) the mathematical rigor of the reasoning can never justify a theory based on postulates if these postulates do not correspond to the true nature of the observed phenomena. The use of even the most sophisticated forms of mathematics can never be considered as a guarantee of quality. (Maurice Allais)

Maurice Allais, 1988, "An outline of my main contributions to economic science."

Nobel lecture

Part I - Introduction to robustness analysis

Some thoughts on robustness (1/2)

- In Operational Research and Decision Analysis, the term **"robustness"** is often used with a different meaning, relating to flexibility, stability, sensitivity, etc.

- Bernard Roy (2010) proposed the term "robustness concern" instead of "robustness analysis", due to the nature of "analysis", which concerns an *a posteriori* process, in contradiction to "concern"

- Robustness analysis is distinguished from sensitivity analysis, which is generally based on perturbations of a single parameter at a time.

Some thoughts on robustness (2/2)

- Robustness is considered as a tool of resistance against the imprecision phenomena, incomplete information and zones of ignorance.

- Robustness primarily concerns the decision model in the light of the claim "robust models produce a fortiori robust results", but also the results and decision support activities (conclusions, documentation, ...).

- Robustness appears as a model that measures and analyzes the gap between the actual model of the Decision Maker and the one resulting from computing devices.

Robustness analysis against "frailty points"

The aim of robustness analysis is to bridge the gap between the Formal Representation (FR) and the real-life context (RLC), in which decisions are made, executed and judged (Roy, 2010). All vague approximations and zones of ignorance, which cause that $FR \neq RLC$ are called **frailty points**, and are categorized as follows:

- The way that imperfect knowledge is treated (ignored, probability distributions, fuzzy numbers, etc.)
- Generation of data from questionable procedures (objective measurements, preferential attributions of meaning, arbitrary shift from a qualitative to a quantitative scale, etc.)
- Modeling of complex aspects of reality, which are difficult to conceive, with the use of parameters (risk attitude of policy makers, trade-offs between economic and environmental impact, etc.)
- The way that technical parameters and rules are introduced to the model (standard deviation thresholds, bounds limiting the domain of investigation, etc.).

Robustness analysis in MCDA

- In multicriteria decision analysis (MCDA) methodological frameworks (outranking methods, analytical methods, disaggregation), robustness analysis mainly focuses on the accuracy/stability of the model's input parameters

- Significant focus on the research and practical field of MCDA, nowadays, is given on the robustness of the **preferential parameters** (i.e. criteria weights), articulated by the Decision Maker (DM).

- It has been evidenced that even the smallest perturbations in the preferential parameters can cause significant alterations to the final decision results (Siskos E. and Psarras J., 2017).

- Robustness should be therefore **measured, analyzed and controlled** in any decision support problem/activity.

Issues related to robustness

- When a decision model could be considered as reliable (analyst's point of view)?
- How can we measure the robustness of a decision model?
- How robustness indicators could be increased?
- Is a decision model acceptable (DM's point of view)?

The need for robustness analysis

- The stability of a model or/and of a solution should be assessed every time
- The analyst should be able to have a clear picture regarding the reliability of the produced results
- Robustness should be expressed using measures which are understandable by the decision maker
- Based on these measures the decision maker may accept or reject the proposed decision model

Part II - Robustness in outranking methods

Robustness in outranking methods

- Outranking methods (i.e. ELECTRE, PROMETHEE, etc.), make use of criteria importance weights
- The evaluation results, and therefore the final decision to be taken, are are highly dependent on these weights
- The elicitation of weights by the decision maker is a difficult procedure, needing careful and rigorous planning

Weights elicitation methods

Direct assessment

- Direct assignment of weights by the DM

Indirect methods

- Pairwise comparisons,
- Simos method (Simos, 1990)
- AHP (Saaty, 1994),
- MACBETH (Bana e Costa and Vansnick, 1997)
- Method of centralized weights (Solymosi and Dombi, 1986)
- Tactic Method (Vansnick, 1986)
- DIVAPIME (Mousseau, 1995)

The Simos method

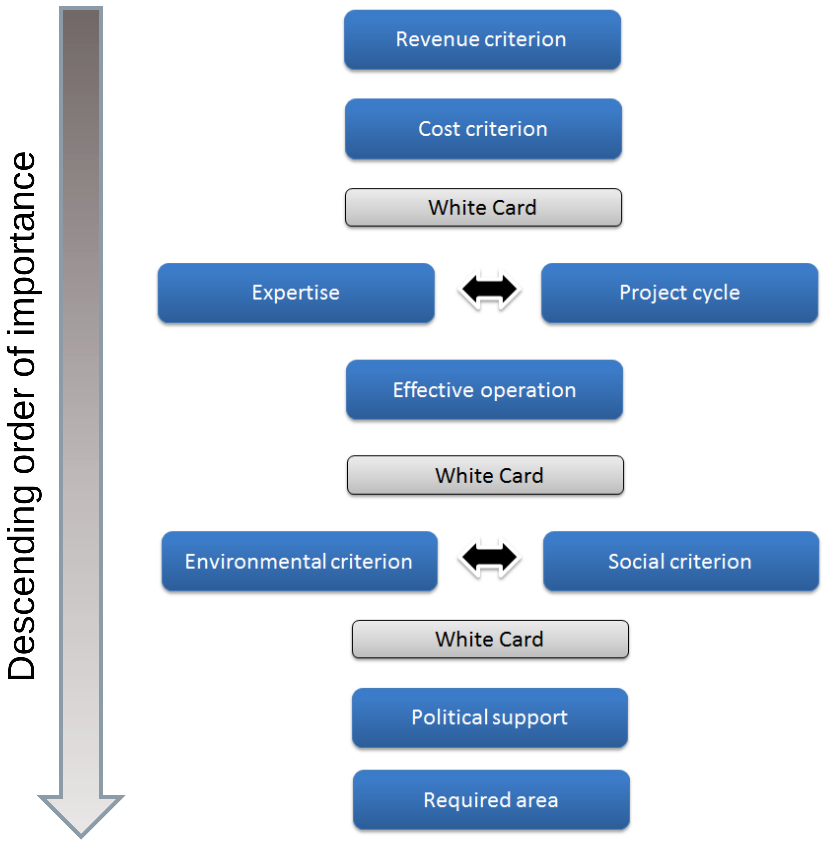
- Simos method
  - Criteria cards
  - White cards
  - Clips

→ Hierarchy

- High potentiality and applicability
  - Energy planning
  - Environmental management
  - Urban planning

An example of the Simos

Criteria Prioritization in a RES investment problem



LP problem solution
(Siskos E. at al., 2014)

SIMOS results

w_3=0,224
w_2=0,172
w_5=0,132
w_6=0,132
w_4=0,112
w_7=0,072
w_1=0,072
w_9=0,052
w_8=0,032

Theoretical concerns
(Siskos E. and Tsotsolas, N., 2015)

- **Proposition 1:**
  The weighting solution of Simos method is a vector of a non-empty convex polyhedral set.

- **Proposition 2:**
  The polyhedral set *P* either contains a single criteria weighting, or an infinite number of weighting vectors that are all consistent with the DM's criteria ranking.

What can we do?

- Figueira and Roy (2002) proposed a revised Simos procedure

They introduced a new preferential parameter, namely the ratio z between the best and worst criterion.

- Further measures are required to ensure the stability of the Simos results (Siskos, E., and Tsotsolas, N. 2015)

*Siskos, E., Tsotsolas, N. (2015). Elicitation of criteria importance weights through the Simos method: A robustness concern, *European Journal of Operational Research*, 246(2), pp. 543-553.

Need for robustness control

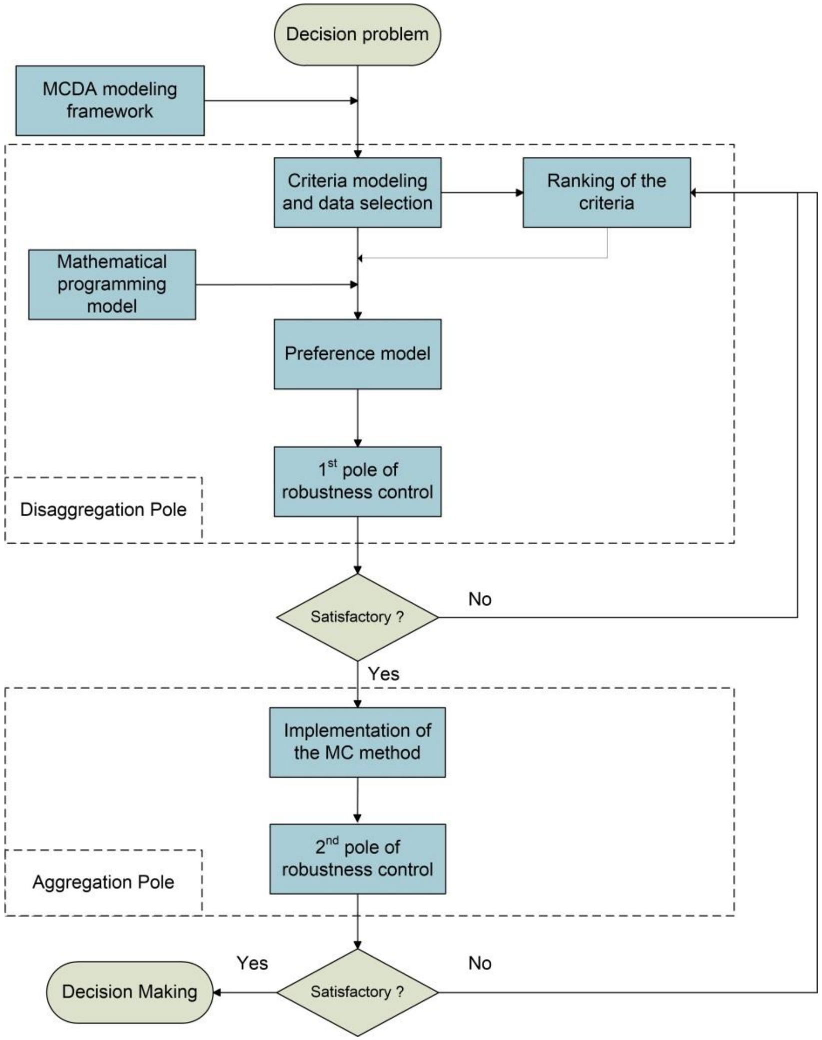
The problem

Implementation of the Simos method → Multiple optimal solutions → Questions with regard to robustness

The proposed approach

- Synergy of the **Simos** and the **PROMETHEE II** methods
- Consideration of multiple criteria weightings
- Assessment and focus on the DM's preferences
- Examination of the robustness of both the **model** and the obtained **results**
- Proposition of indices to measure robustness

Proposed robustness control procedure



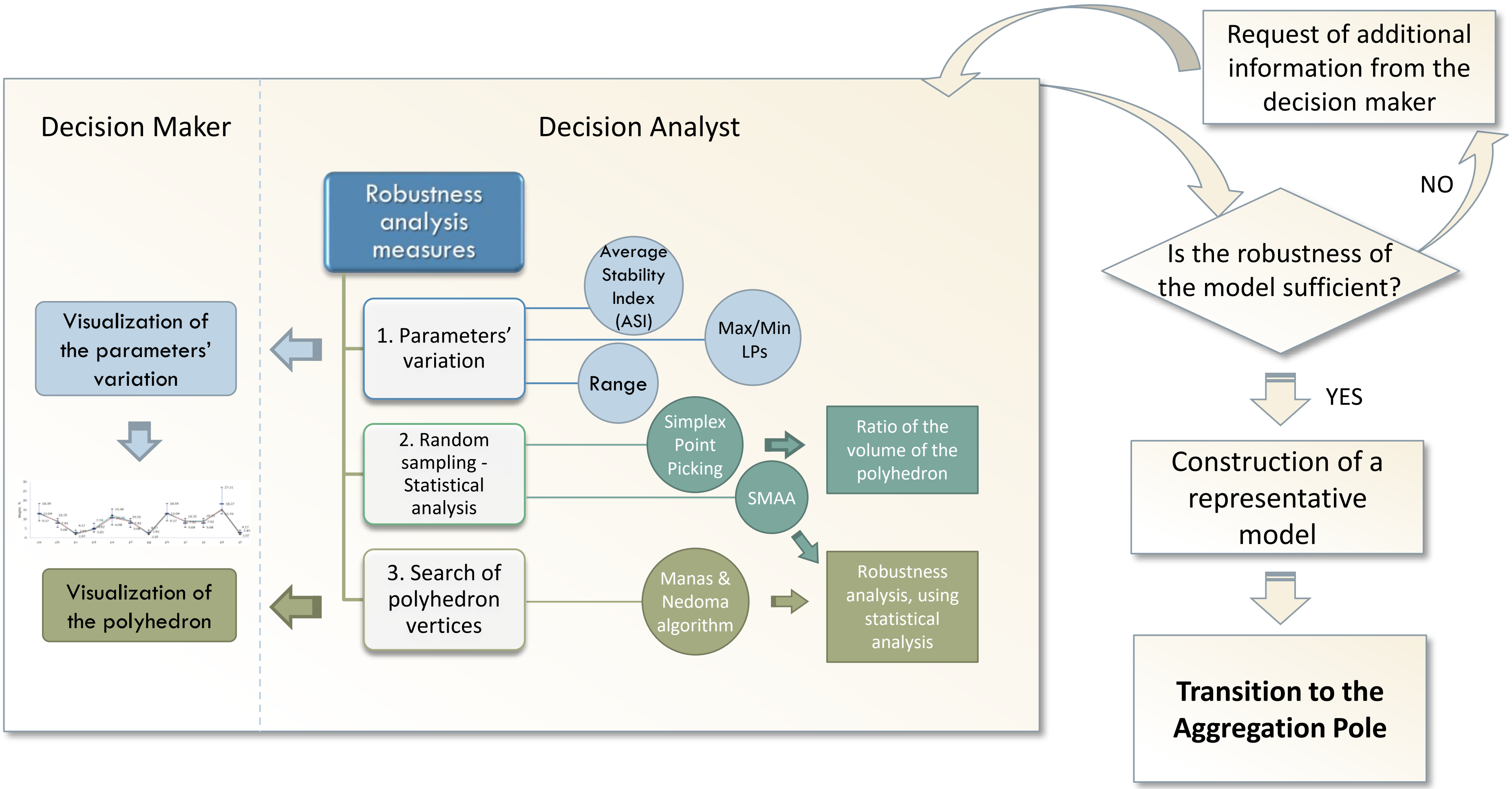
Special features of the two poles

- Depending on the nature of each evaluation problem, robustness control can be used in a single pole, instead of both.

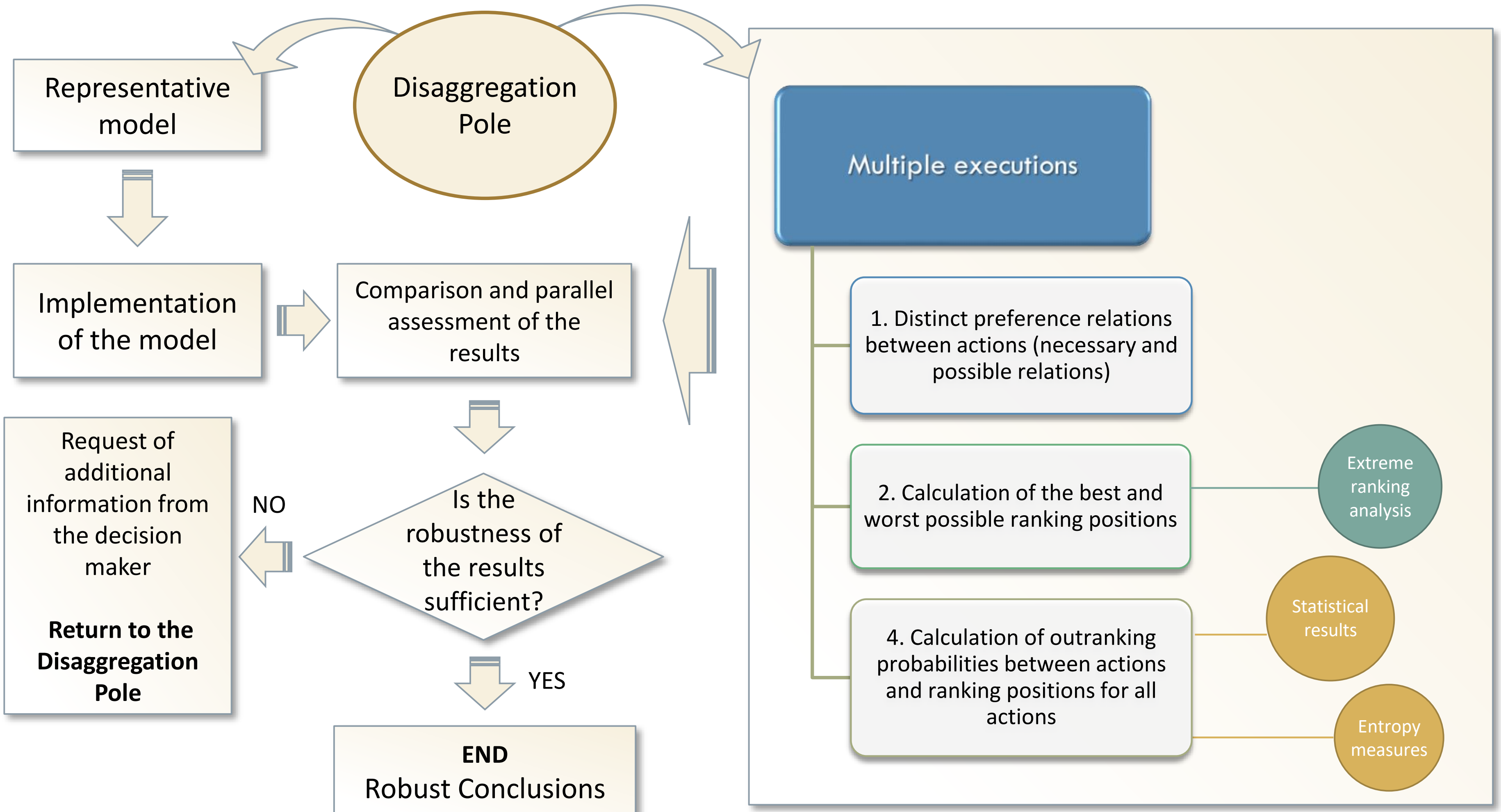
- Each pole with each own characteristics can provide the analyst with the desirable robustness measurements and assessments.

Feature	Disaggregation pole	Aggregation pole
Aim	Modeling support	Decision support
Actions involved	Reference alternatives	Real world actions
Preference elicitation	Criteria ranking, pairwise comparisons	Complete ranking
DM's involvement	Strong	Weak
DA's involvement	Strong	Strong
Robustness analysis	Yes	Yes
Robustness techniques and indices used	ASI, Parameters' variation, Visualization, Volume of the feasible space	Extreme Ranking Analysis, Visualization, Rank probabilities, Entropy measures

Disaggregation Pole
1st Control Point



Aggregation Pole
2nd Control Point



Robustness Indices

(Matsatsinis N., Grigoroudis, E. and Siskos, E., 2018)

Disaggregation pole

- They focus on the efficacy/stability of the model to produce results that are stable and not misleading or ambiguous
- They monitor that the decision model under construction accurately reflects the preferences of the DM
- They prevent the analyst from performing heavy, pointless computations, which are certain to reach results of low quality

Aggregation pole

- They are also necessary, since good indices in the 1st pole, do not guarantee robust evaluation results in the end
- They are usually coupled with certain techniques and methods
- They are assessed by the DM prior to reaching his/her final decision

Robustness Indices (1st pole)

Average range of the preferential parameters (ARP)

Calculated after following a min/max procedure of all preferential parameters

$$ARP = \frac{1}{S}\sum_{s=1}^{S}\left[\max_r(p_{rs}) - \min_r(p_{rs})\right]$$

ARP ranges in [0, 1]. It receives **lower values**, when the robustness of a model increases

Average Stability Index (ASI)

$$ASI = 1 - \frac{1}{n}\sum_{i=1}^{n}\frac{\sqrt{m\sum_{j=1}^{m}p_{ij}^2 - \left(\sum_{j=1}^{m}p_{ij}\right)^2}}{\frac{m}{n}\sqrt{m-1}}$$

ARP ranges in [0, 1]. It receives **higher values**, when the robustness of a model increases

Robustness Indices (2nd pole)

Average range of ranking (ARRI)

*Calculated after implementing the Extreme Ranking Analysis**

$$ARRI = \frac{1}{m}\sum_{k=1}^{m}\left(\left|R_*(k) - R^*(k)\right| + 1\right)$$

ARRI ranges in [0, 1]. It receives **higher values**, when the robustness of a model increases

Ratio of the average range of ranking (RARR)

$$RARR = \frac{ARRI - 1}{m - 1} \cdot 100\%$$

ARRI ranges in [0%, 100%]. It receives **lower values**, when the robustness of a model increases. Optimal value is 0%.

*Kadziński et al. (2012)

Robustness Indices (2nd pole)

Statistical Preference Relations Index (SPRI)

Calculated after implementing any random sampling technique, or any method that generates a statistically adequate number of preferential parameters vectors

$$SPRI = \frac{1}{R}\sum_{k=1}^{m}\sum_{t=1}^{m} P_t^k$$

The probability that an alternative a_k gets ranked in the t-th position is:

$$P_t^k = \frac{c_t^k}{R} \cdot 100\%$$

SPRI ranges in [0%, 100%]. It receives **higher values**, when the robustness of a model increases. Optimal value is 100%.

Extreme Ranking Analysis (Kadzinski et al., 2012)

- It is a newly proposed analysis that estimates each alternative's best and worst possible ranking positions.
- It uses special linear programming techniques.

Best possible rank of alternative A

N^{*}_{A}: The number of alternatives that **surpass alternative A** in the ranking under any circumstances.

UTA II constraints

+

$$U(a) \geq U(b) - Mu_b \forall b \in A \setminus \{\alpha\}$$

$$[min] F = \sum_{b \in A \setminus \{a\}} u_b$$

M: auxiliary variable equal to a big positive value

u_b : an integer variable

Worst possible rank of alternative A

N^{*}_{A}: The number of alternatives that **are surpassed by alternative A** in the ranking under any circumstances.

UTA II constraints

+

$$U(b) \geq U(a) - Mu_b \forall b \in A \setminus \{\alpha\}$$

$$[min] F = \sum_{b \in A \setminus \{a\}} u_b$$

M: auxiliary variable equal to a big positive value

u_b : an integer variable

Part III - Application to e-gov evaluation

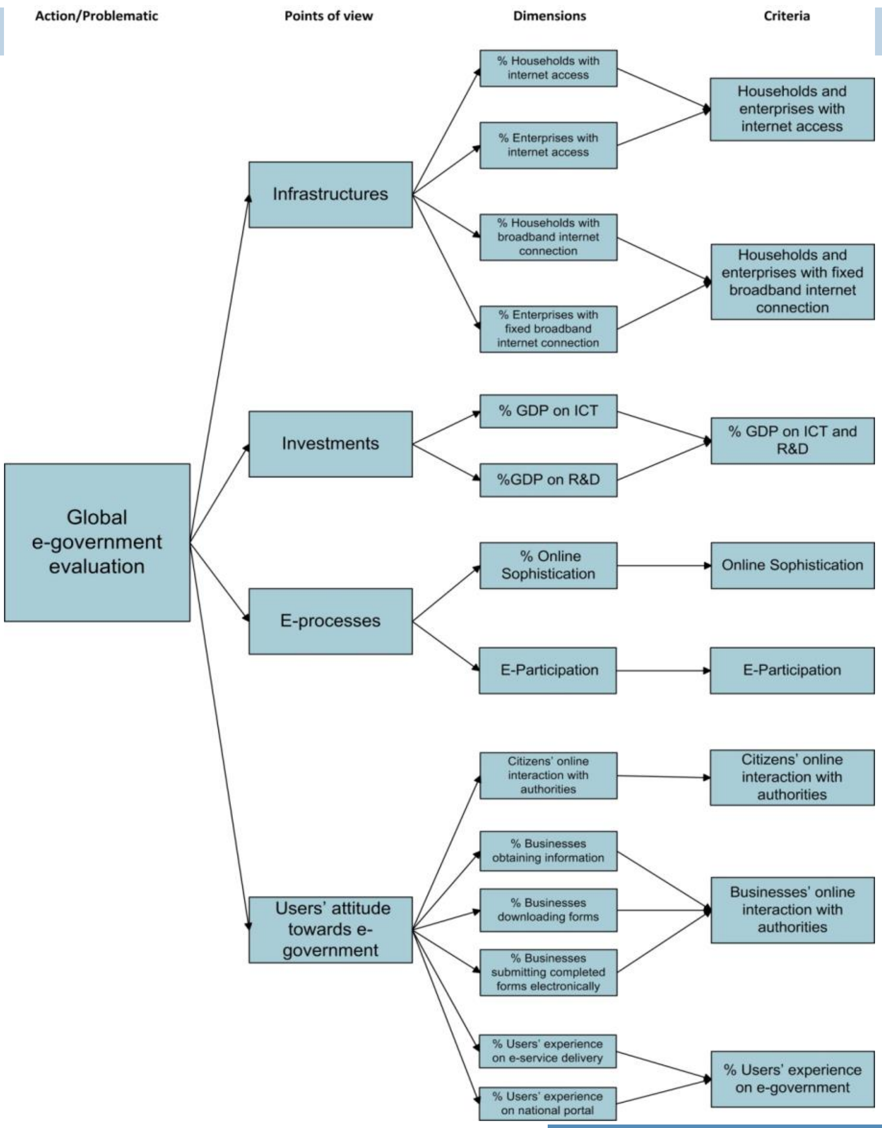
E-government

- Use of Information and Communication Technology in Public Administration and Local Government, in order to digitally provide more **efficient**, more **qualitative** and more **secure** services to citizens and businesses.
- E-government refers to the use of electronic media to achieve:
  - Interaction between governmental entities and citizens, governmental entities and businesses.
  - Simplification and efficiency of internal governmental functions.
- It aims to simplify and improve democratic, governmental and business aspects of Governance (Dawes, 2008).
- **The aim of the research agenda** is to develop an evaluation model to rank the European countries based on their e-government development

E-government evaluation system
(Siskos, Askounis & Psarras, 2016)

22 European Countries

Belgium
Czech Republic
Denmark
Germany
Estonia
Ireland
Greece
Spain
France
Italy
Hungary
Netherlands
Austria
Poland
Portugal
Slovenia
Slovakia
Finland
Sweden
Norway
United Kingdom
Croatia



Consistent family of criteria

1. % access to the web "g_1"
2. Broadband and mobile (3G-4G) internet connection "g_2"
3. % GDP investment on ICT and R&D "g_3"
4. Online Sophistication "g_4"
5. E-participation "g_5"
6. % Citizens' online interaction with authorities "g_6"
7. % Businesses' online interaction with authorities "g_7"
8. Users' experience "g_8"

Eight evaluation criteria (g_1-g_8)

Criterion	Index	Worst level	Best level
g_1	% population	0	100
g_2	% population	0	100
g_3	% GDP	0	5
g_4	Index %	0	100
g_5	Index [0-1]	0	1
g_6	% citizens	0	100
g_7	% businesses	0	100
g_8	Index %	0	100

Criteria indices and evaluation scales

Multicriteria evaluation framework

1. Ranking of the criteria and estimation of the weights using the Simos method

- Ranking of the criteria by the DM
- Additional pairwise comparisons / aspiration levels needed

2. Application of the PROMETHEE II method for the evaluation and ranking of the alternatives

- Calculation of the PROMETHEE II net flows, based on which the countries are ranked

3. Robustness control procedure

- Measurement of the model's robustness (1st pole)
- If satisfactory, the procedure moves to the aggregation pole (2nd pole), where the robustness of the results is assessed

Estimation of the criteria weights, based on the Simos method

Ranking of the criteria on descending preference
g_2
White card
g_6
g_3
g_1
g_4
White card
g_8
g_5, g_7

Implementation of the Simos method, without considering robustness

(Siskos and Tsotsolas, 2015)*

→

Criteria weights

$p_2 = 0.24$
$p_6 = 0.19$
$p_3 = 0.17$
$p_1 = 0.14$
$p_4 = 0.12$
$p_8 = 0.07$
$p_5 = 0.04$
$p_7 = 0.04$

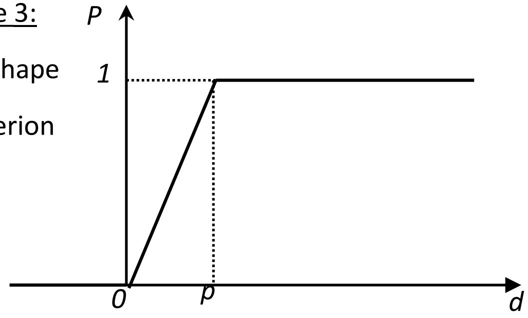
PROMETHEE II modeling framework

Countries	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
BE	90	67.8	2.28	63	0.25	36	74.33	58
CZ	88	63.8	1.91	56	0.25	21.33	87.67	44.5
DN	96	72.8	3.05	85	0.55	65.33	89.33	65
GE	93.5	71.3	2.94	67	0.7	33.33	58.67	45.5
EE	89.5	69.5	1.74	87	0.76	35	80	77.5
IR	90	66.2	1.58	87	0.65	43	88	62
GR	77.5	59.6	0.78	46	0.8	27.67	78.33	41
ES	86	66.4	1.24	91	0.78	36.33	69	72.5
FR	91	68.5	2.23	75	0.96	44	89	68.5
HR	82	61.7	0.81	53	0.33	19.33	81	48
IT	85.5	63.4	1.25	77	0.78	15.67	69.67	60.5
HU	81.5	62.0	1.41	45	0.45	34.33	82.33	35.5
NL	98	77.4	1.98	82	1	57.67	80.67	65.5
AT	89.5	67.4	2.81	86	0.63	40.33	80.67	70.5
PO	84	62.5	0.87	76	0.49	17.33	81.67	51
PT	81	63.6	1.36	96	0.65	30.67	81	74
SLO	87.5	67.4	2.59	68	0.39	37	85	63
SLK	88	63.5	0.83	72	0.63	33	80.67	30
FI	95	75.5	3.32	86	0.71	64	91.33	71
SE	94	74.9	3.21	83	0.61	60.33	90.67	68.5
NO	95	71.4	1.69	78	0.69	64.33	84.33	63.5
UK	92.5	72.3	1.63	74	0.96	35	75	51
Preference Thresholds	*5.0*	*5.0*	*0.10*	*10*	*0.10*	*10.00*	*5.00*	*10.0*

Type 3:

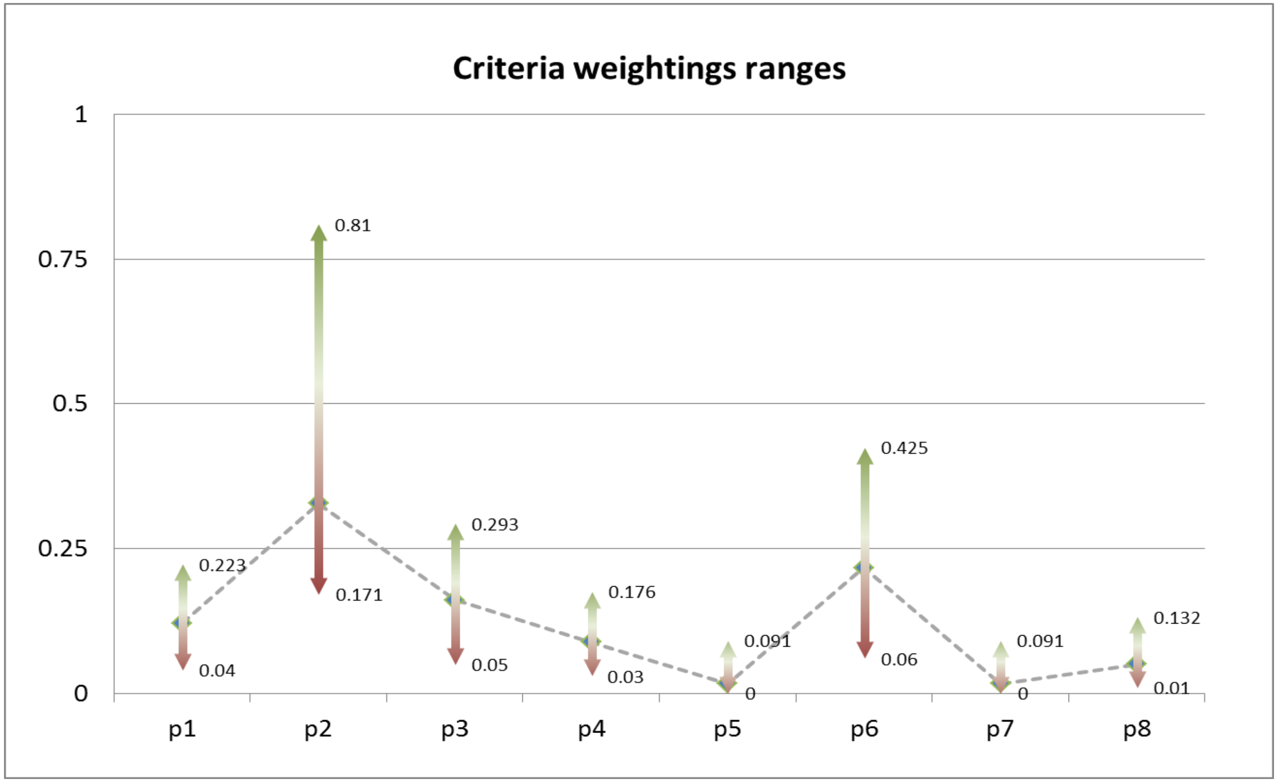
V - shape

Criterion



$$P(d) = \begin{cases} 0 & d \leq 0 \\ \dfrac{d}{p} & 0 \leq d \leq p \\ 1 & d > p \end{cases}$$

Robustness analysis - Round 1



ASI
0.906

ARP INDEX
23.5 %

- Examination of the Average Stability Index (ASI) and the Average Range of the preferential Parameters (ARP).
  (Matsatsinis, N.F., Grigoroudis, E., Siskos, E. 2018)

Robustness analysis - Round 2

- Definition of a minimum threshold for the importance of the two least important criteria at 3%, by the DM

ASI
0.937

ARP INDEX
15.8 %

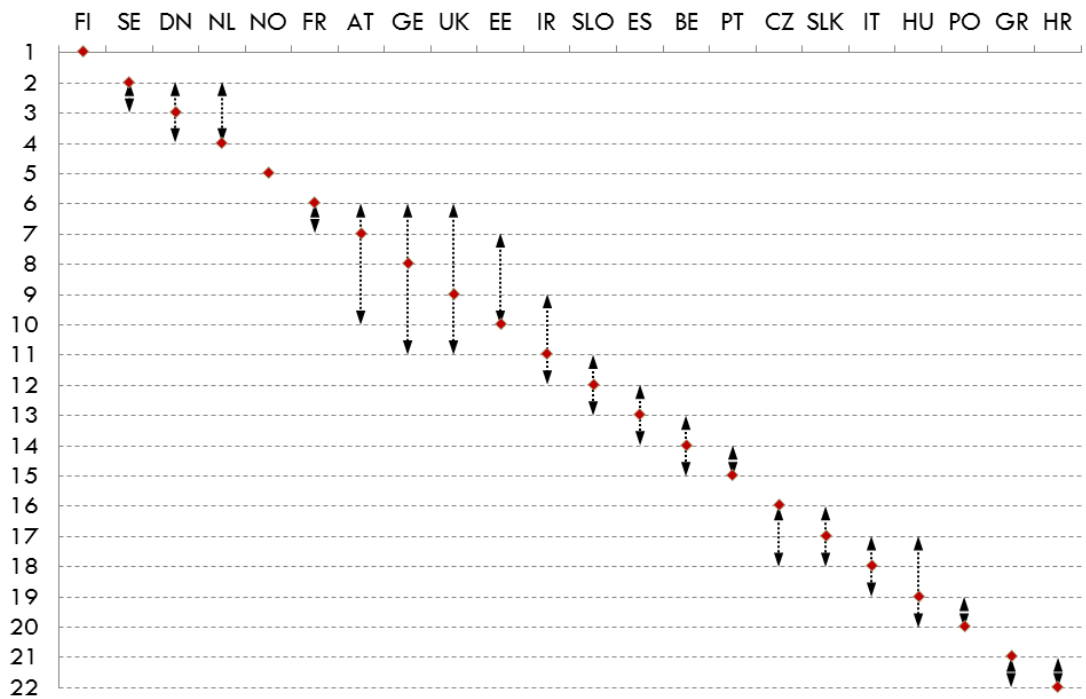
- Both robustness indices increase significantly
- This permits the analyst to forward the robustness control procedure to the aggregation pole

Round 2 – Aggregation pole

Representative ranking

Rank	Country	Net Flow
1	FI	0.806
2	SE	0.717
3	DN	0.689
4	NL	0.666
5	NO	0.450
6	FR	0.270
7	AT	0.230
8	GE	0.211
9	EE	0.180
10	UK	0.170
11	IR	0.070
12	SLO	0.032
13	BE	0.003
14	ES	-0.085
15	PT	-0.282
16	CZ	-0.424
17	SLK	-0.446
18	IT	-0.524
19	HU	-0.549
20	PO	-0.633
21	HR	-0.763
22	GR	-0.787

Implementation of the Extreme Ranking Analysis



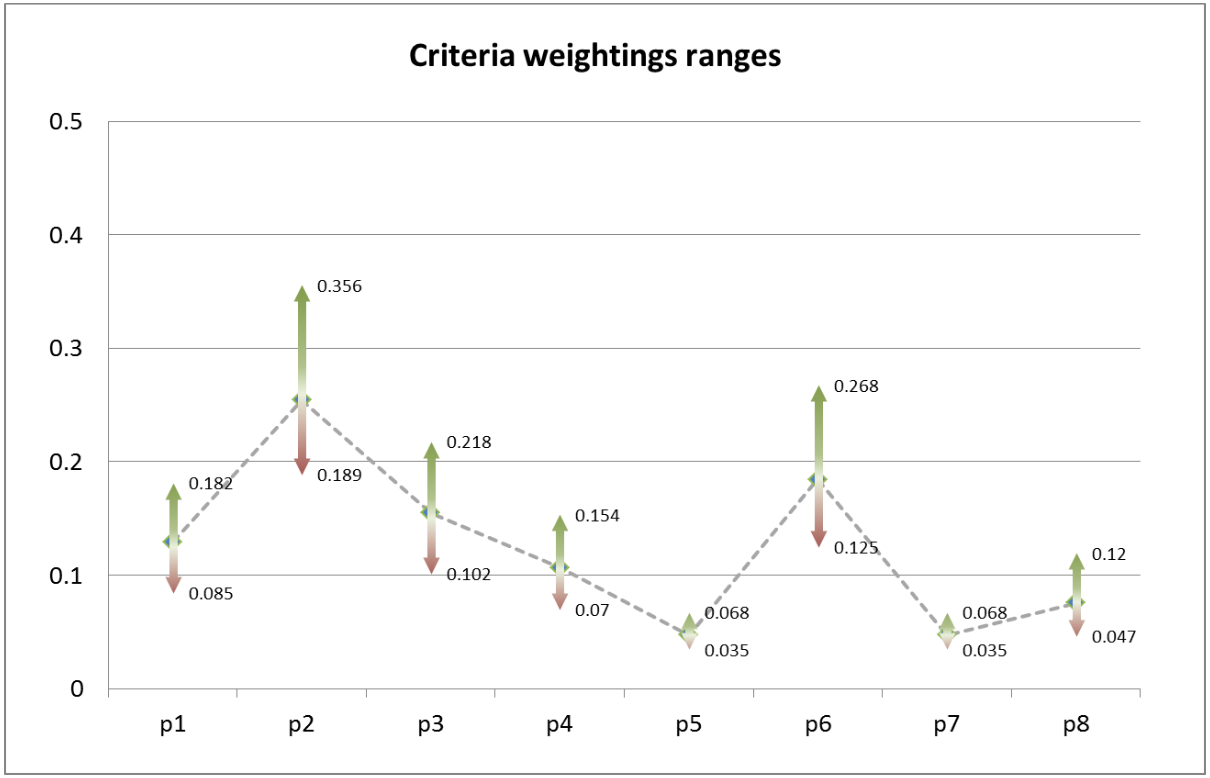
Calculation of the Average Range of the Ranking Index (ARRI) and its ratio RARR

$ARRI = 3.0$

$RARR = 9.7\%$

Robustness analysis - Round 3

- Articulation of an importance ratio between the most and the least important criteria.
- The ratio z is defined in-between the range [5.0 - 5.5]



ASI
0.964

ARP INDEX
9.3 %

- The robustness indices again increase significantly
- Robustness control safely moves to the aggregation pole

Round 3 – Aggregation pole

Representative ranking

Rank	Country	Net Flow
1	FI	0.806
2	SE	0.717
3	DN	0.689
4	NL	0.666
5	NO	0.450
6	FR	0.270
7	AT	0.230
8	GE	0.211
9	EE	0.180
10	UK	0.170
11	IR	0.070
12	SLO	0.032
13	BE	0.003
14	ES	-0.085
15	PT	-0.282
16	CZ	-0.424
17	SLK	-0.446
18	IT	-0.524
19	HU	-0.549
20	PO	-0.633
21	HR	-0.763
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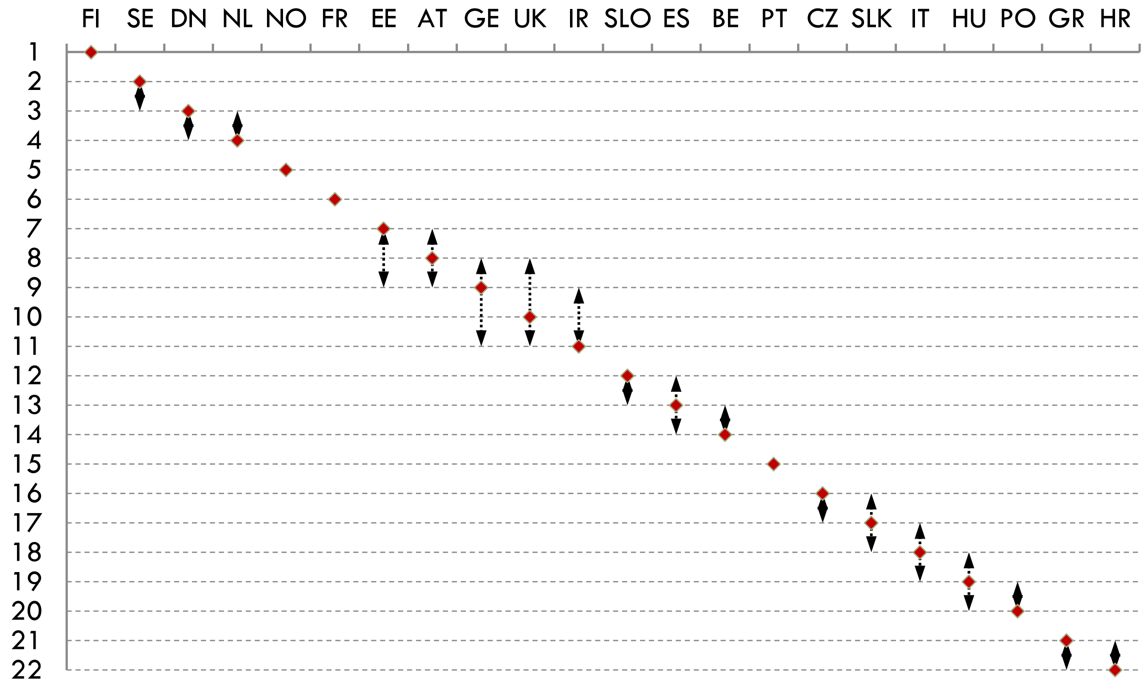
Implementation of the Extreme Ranking Analysis



$ARRI = 2.3$

$RARR = 6.2\%$

Evolution of the robustness indices

- **Disaggregation pole indices**

Index	Round 1		Round 2		Round 3
ASI	0.906	→ +3.4%	0.937	→ +2.9%	0.964
ARP	23.5%	→ -32.7%	15.8%	→ -41.1%	9.3%

- **Aggregation pole indices**

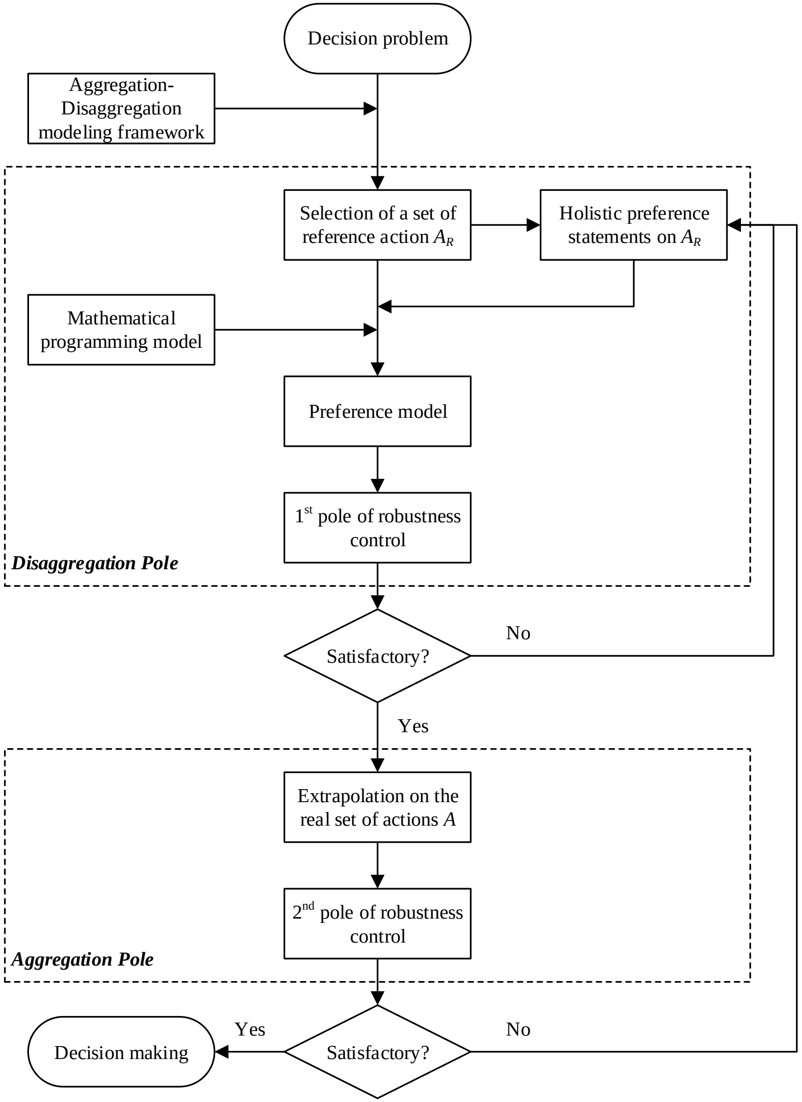
Index	Round 2		Round 3
ARRI	3.0	→ -23.3%	2.3
RARR	9.7 %	→ -36.1%	6.2 %

Part IV - Robustness in disaggregation methods

Robustness in disaggregation methods (UTA family, etc.)

- Disaggregation methods (i.e. UTA I, UTASTAR, etc.), make use of value functions, elicited by the DM in an implicit way
- Any UTA-type inference engine shows that the DM's preference model may not be a unique additive value function
- In contrary, there exists an infinite set of different functions, all being compatible with the holistic preference statements of the DM
- Robustness of the decision model and the obtained results should therefore be examined, analyzed and controlled

Proposed robustness control procedure



Need for robustness control

The problem

Solution of the UTA LP problem → Multiple optimal solutions → Questions with regard to robustness

The proposed approach

- Synergy with disaggregation methods and ROR techniques
- Consideration of multiple value functions
- Assessment and focus on the DM’s preferences
- Examination of the robustness of both the **model** and the **results**
- Proposition of indices to measure robustness

Part V - Application to job evaluation

Problem presentation

- Real world job evaluation problem in a leading Greek organization
- Job evaluation is a systematic process that enables the design and establishment of human resources improvement procedures and fair reward systems
- This application concerns the assessment of a value system that encapsulates the importance of the parameters that reflect the global responsibility and duties of each different job position
- The evaluation does not concern the real persons in these positions, but the jobs themselves

Evaluation system

- *Criterion 1* (input criterion): Required qualifications and skills (i.e., basic knowledge, expertise, skills, experience)
- *Criterion 2* (process criterion)Contribution to decision making (e.g., participation to committees, position in the hierarchy, problem solving, quantity and importance of the decisions)
- *Criterion 3* (output criterion): Responsibility (e.g., qualitative results, geographical area, degree of responsibility, perspectives, strategic role in development activities, and support to other units), measured using an ordinal scale

g_1: Required qualifications and skills	Input	Measurable	Numerical scale [5, 20]
g_2: Contribution to decision making	Process	Ordinal	[limited, medium, high, very high]
g_3: Responsibility	Output	Ordinal	[limited, medium, high, very high]

The alternatives under evaluation

Job position	Criterion 1 (Required qualifications and skills)	Criterion 2 (Contribution to decision making)	Criterion 3 (Responsibility)
A	7	medium	high
B	12	high	medium
C	15	limited	limited
D	5	medium	medium
E	10	limited	very high
F	19	very high	limited
G	12	limited	high
H	8	high	high
I	16	limited	medium
J	6	medium	very high

Preference elicitation (1st iteration)

- Dialogue with the decision maker, in order to rank 4 reference alternatives; one real and three fictitious, according to the UTASTAR rationale.

Reference job position	Criterion 1 (Required qualifications and skills)	Criterion 2 (Contribution to decision making)	Criterion 3 (Responsibility)	Ranking position
Z_1	5	high	very high	1
E	10	limited	very high	2
Z_2	10	high	high	2
Z_3	15	medium	medium	4

Robustness analysis - Round 1

- The implementation of the UTASTAR procedure reveals results of significantly low quality with regard to their robustness; no decision on the ranking of the ten job positions can therefore be supported at this current stage of the analysis
- The *ASI* index takes the value of 0.733, while the average range of the preferential parameters (*ARP*) is 0.686 (i.e., equals to 68.6% of the whole possible ranging area)
- Certain parameters, such as w_{13}, w_{23}, and w_{31} can range from 0 to 0.9, being in essence uncontrollable
- Consequently, the bipolar robustness control procedure does not allow us to move to the aggregation pole (2^{nd} pole of robustness control).

Preference elicitation (2^{nd} iteration)

- Return to the DM, to ask for additional preference information

Dialogue excerpt:

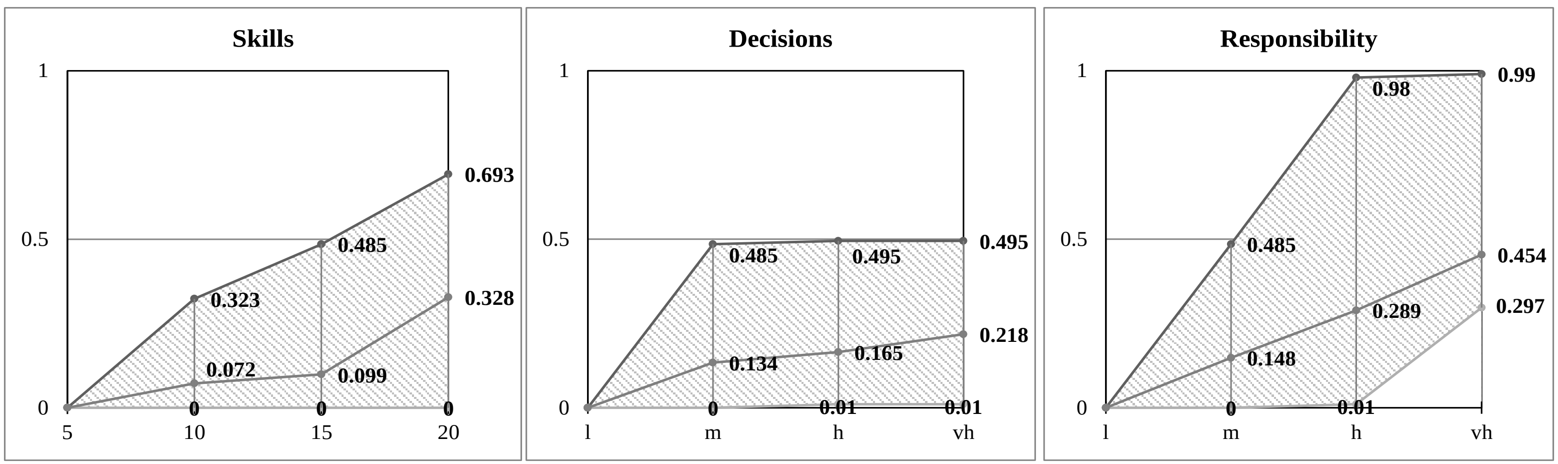
Analyst: It seems that the mathematical input required by the method is not sufficient for a good assessment of your preference model. Would you please insert to your ranking a highly qualified job (17 points), named Z4, with a “very high” contribution to the decision making processes but without any important responsibility (limited)?

DM: I would rate this job fourth, between Z2 and Z3.

- Due to the increasing number of reference alternatives, the analyst decided to decrease and stabilize the value of δ to 0.01.

Robustness analysis - Round 2

- The solution of the LP shows that the DM's ranking is cohesive
- The min/max procedure of the disaggregation pole produces the following visualized results



ASI
0.772

ARP INDEX
56.7%

- Again the analysis cannot proceed to the aggregation pole

Preference elicitation (next iterations)

- Assignment of values to certain pairs of fictitious alternatives by the DM, so that he/she is indifferent between them

Example:

Job position	Criterion 1 (Required qualifications and skills)	Criterion 2 (Contribution to decision making)	Criterion 3 (Responsibility)
Y_5	?	medium	high
Y_6	5	high	very high

Dialogue excerpt:

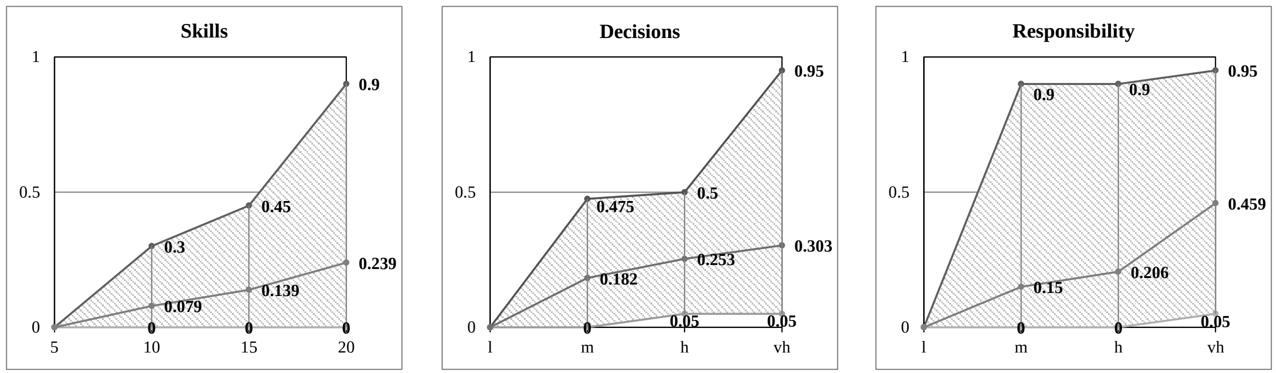
Analyst: Comparing Y_5 and Y_6 what qualification degree (if any) is required for the job Y_5 to compensate exactly the difference in the other two criteria?

DM: I would say the perfect score of 20.

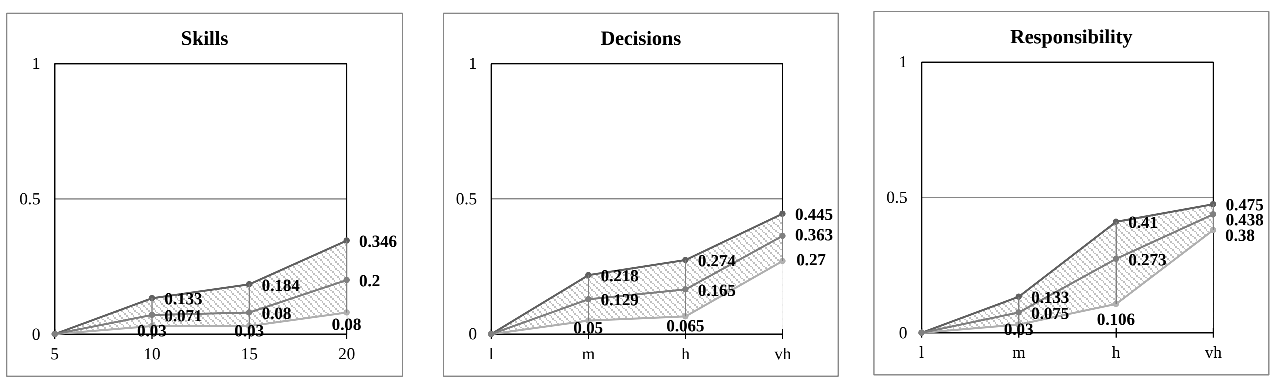
Evolution of the robustness results

- Preferential parameters variation

1st round



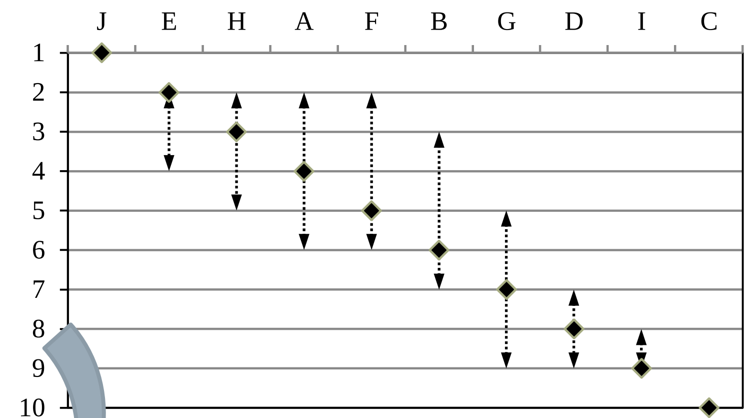
6th round (Final)

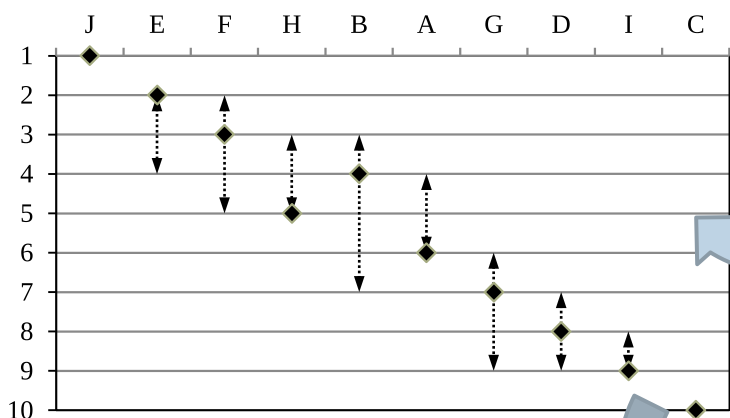


Evolution of the robustness results

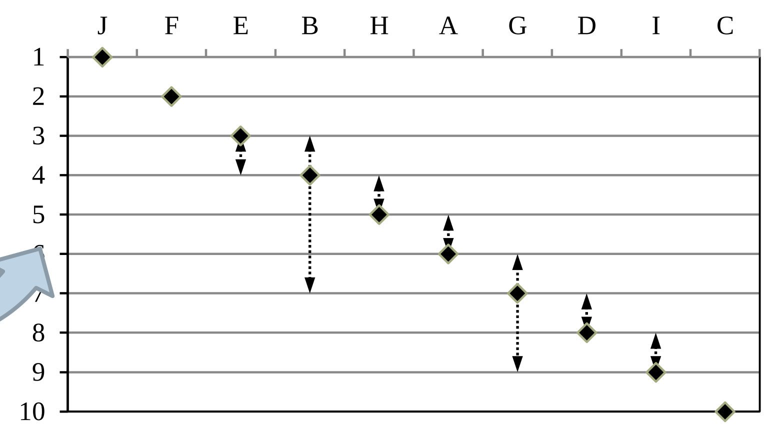
- Extreme ranking results

4th round





5th round



6th round

Evolution of the robustness indices

- In the parenthesis, the percentage of amelioration, after each round, is presented

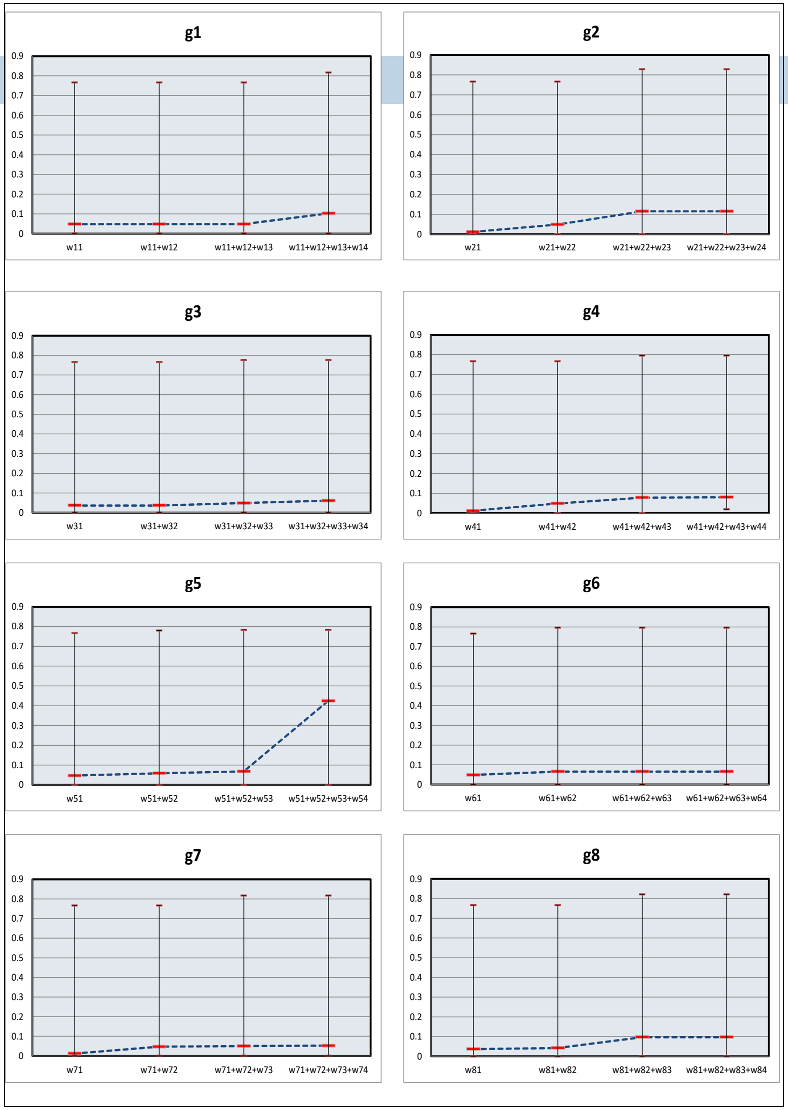
	Disaggregation pole		Aggregation pole	
Iteration	**ARP**	**ASI**	**ARRI**	**RARR**
1	0.686	0.733	-	-
2	0.567 (17.3%)	0.772 (5.3%)	-	-
3	0.427 (24.8%)	0.795 (3.0%)	-	-
4	0.370 (13.3%)	0.808 (1.6%)	3.4	26.7%
5	0.214 (42.2%)	0.898 (11.1%)	2.9 (14.7%)	21.1% (21.0%)
6	0.175 (18.1%)	0.909 (1.2%)	2.2 (24.1%)	13.3% (37.0%)

Part VI – Revisiting e-gov evaluation

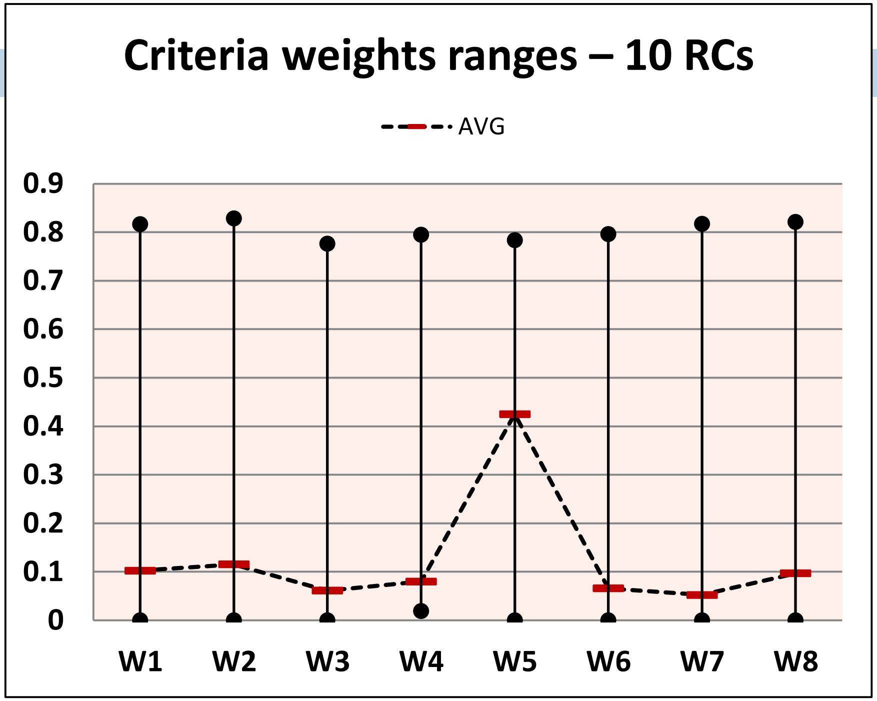
Multicriteria evaluation framework

1. Reference actions/countries
   - Representative of the real actions
   - Ranking of the reference actions by the DM
2. Application of UTASTAR for the assessment of the value functions parameters
   - Minimization of the under/over-estimation errors
3. Robustness control procedure
   - Measurement of the model's robustness (1st pole)
   - If satisfactory, additive value model is applied multiple times
   - Measurement of the robustness of the results (2nd pole)

A’ phase – 10 reference countries

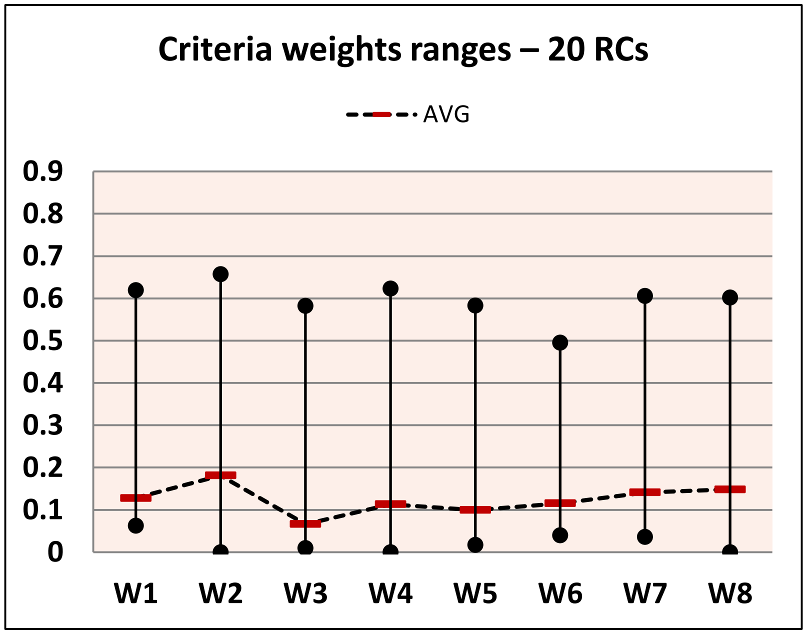


Marginal value functions

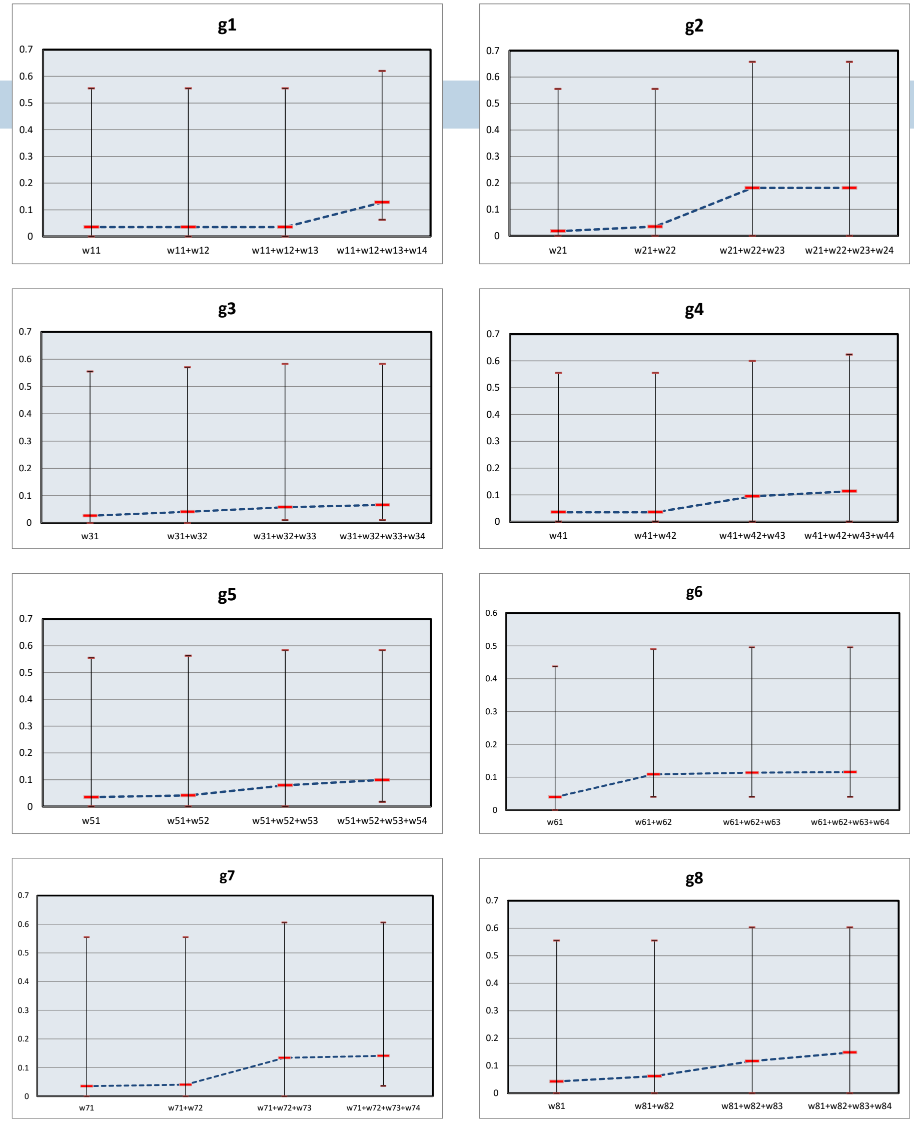


ASI
0.825

B' phase – 20 reference countries



ASI
0.936

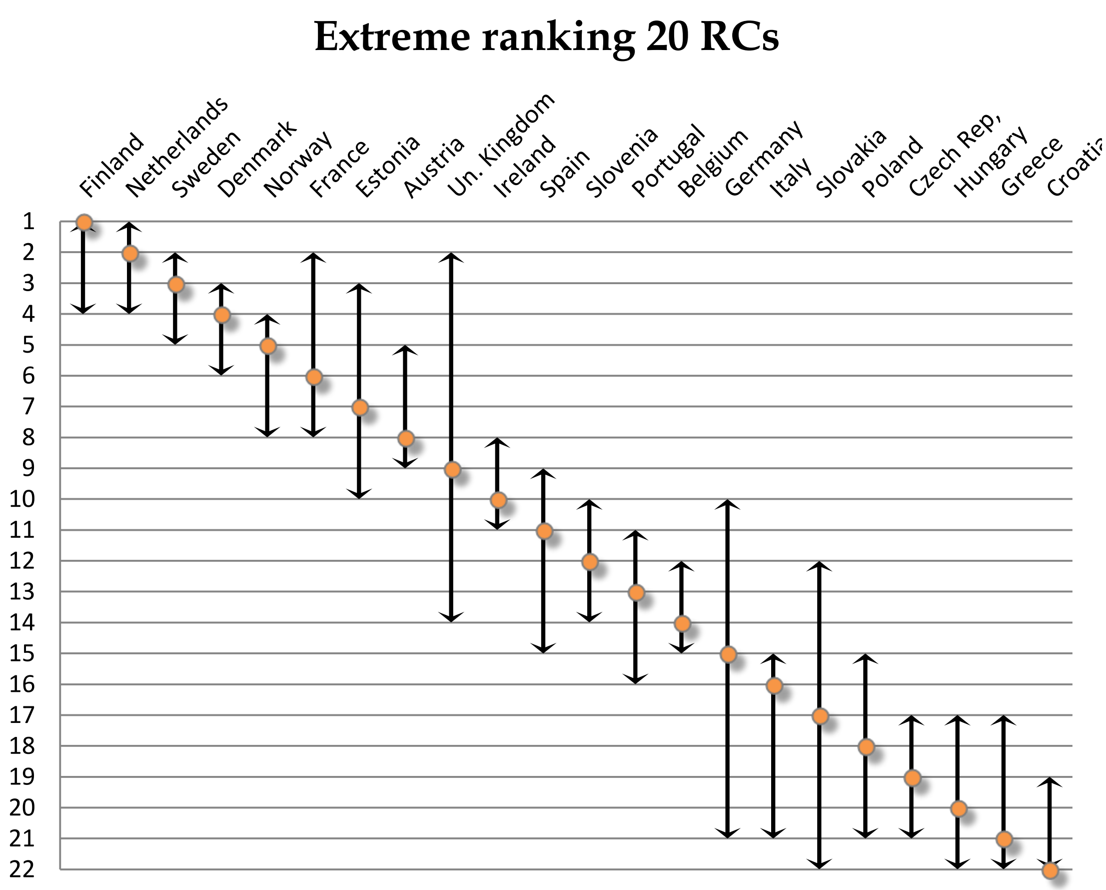


Marginal value functions

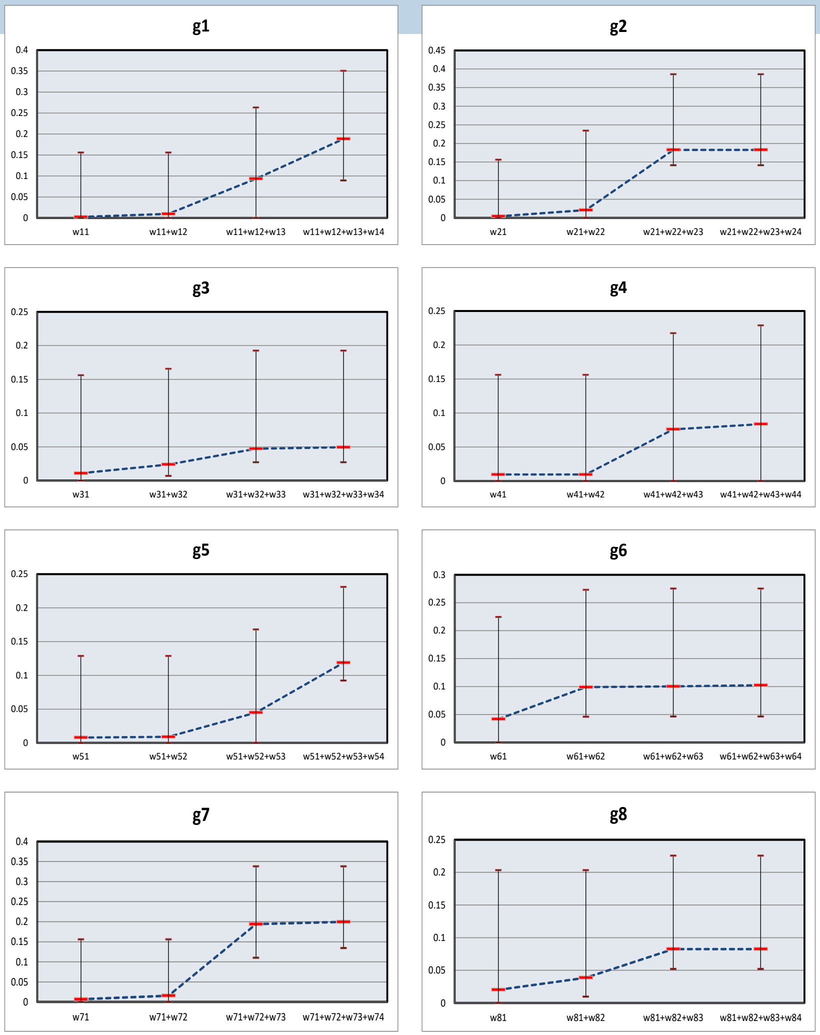
B′ phase – Aggregation pole

Representative ranking

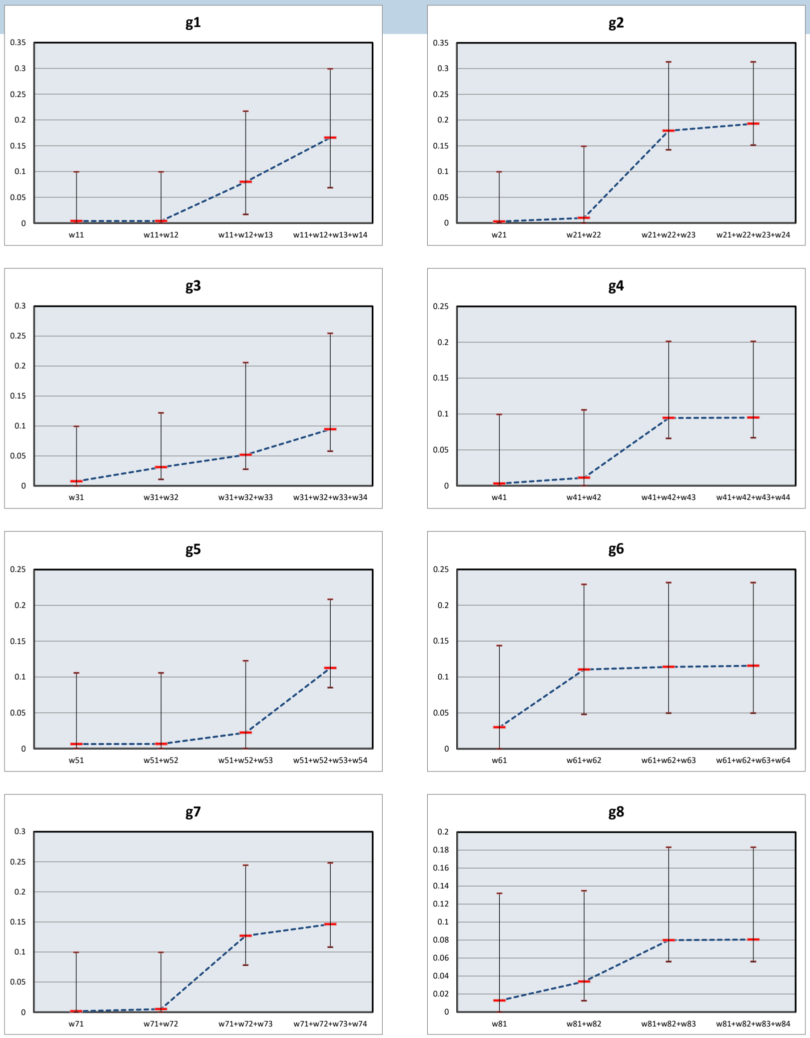
Rank	Country	Value
1	Finland	0.88412
2	Netherlands	0.88303
3	Sweden	0.85519
4	Denmark	0.83486
5	Norway	0.81202
6	France	0.80468
7	Estonia	0.78001
8	Austria	0.76750
9	United Kingdom	0.75750
10	Ireland	0.73773
11	Spain	0.71744
12	Slovenia	0.68527
13	Portugal	0.68365
14	Belgium	0.67066
15	Germany	0.65976
16	Italy	0.61951
17	Slovakia	0.61024
18	Poland	0.56399
19	Czech Republic	0.55190
20	Hungary	0.52388
21	Greece	0.51018
22	Croatia	0.49171



C & D phases – 25 & 30 RCs



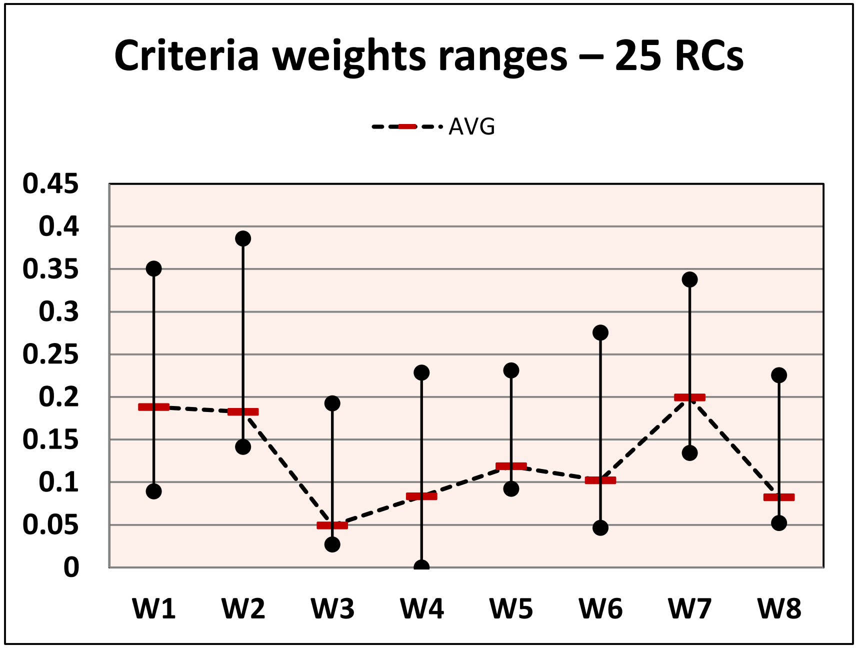
25 RCs

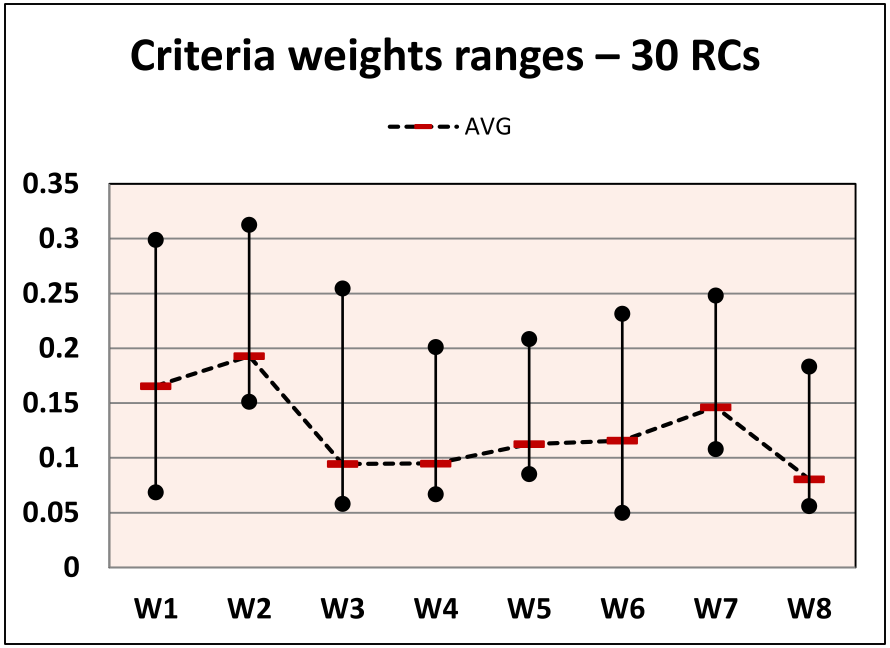


30 RCs

Marginal value functions

C & D phases – 25 & 30 RCs

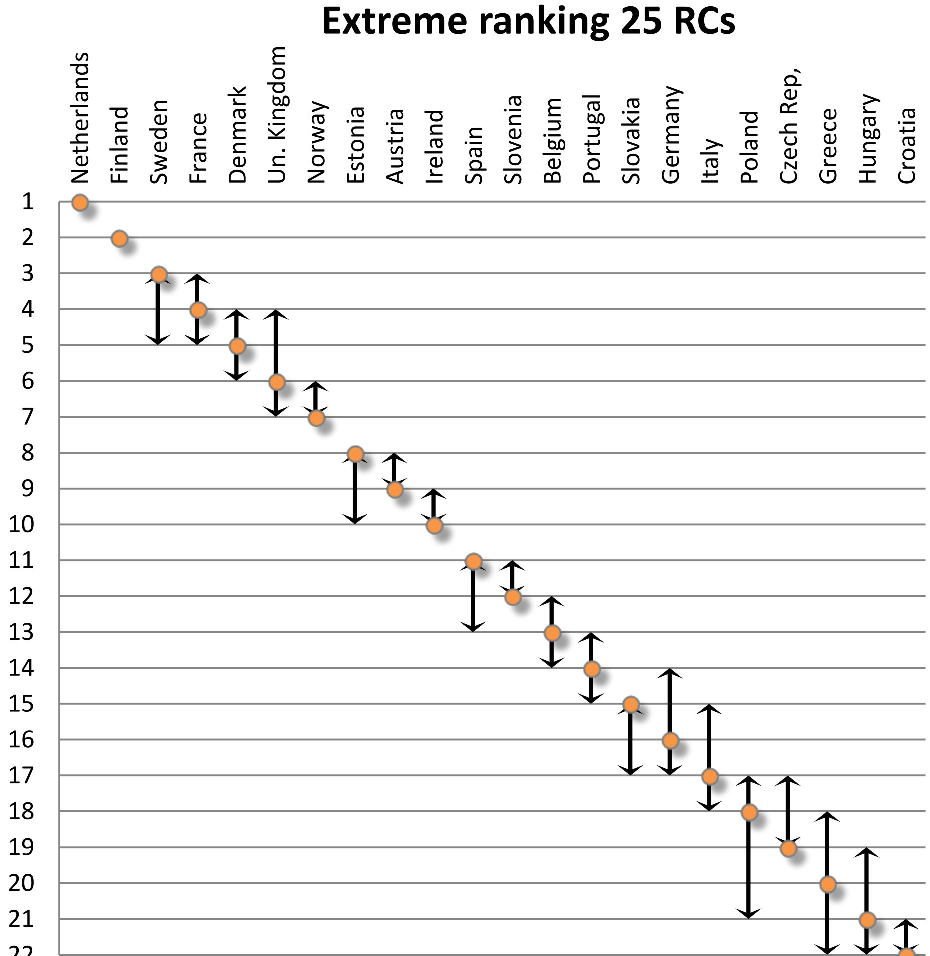


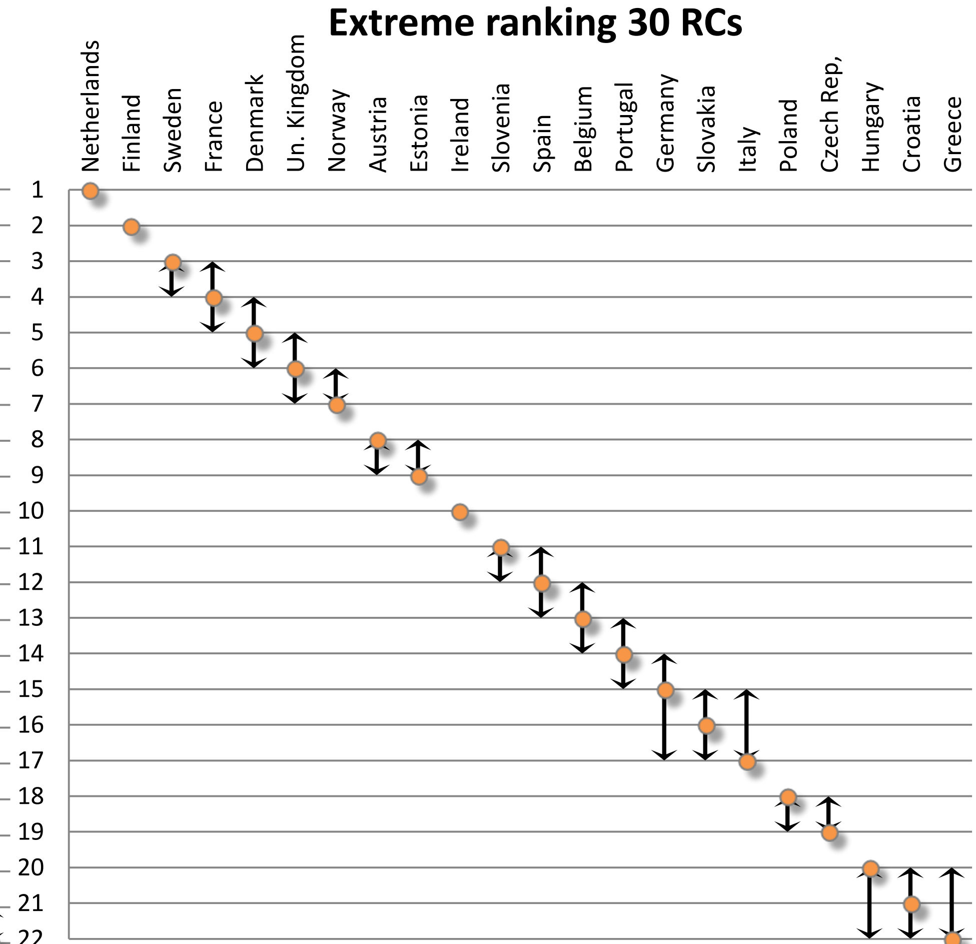


ASI
0.971

ASI
0.980

C & D phases – Extreme Ranking





$ARRI = 2.95$

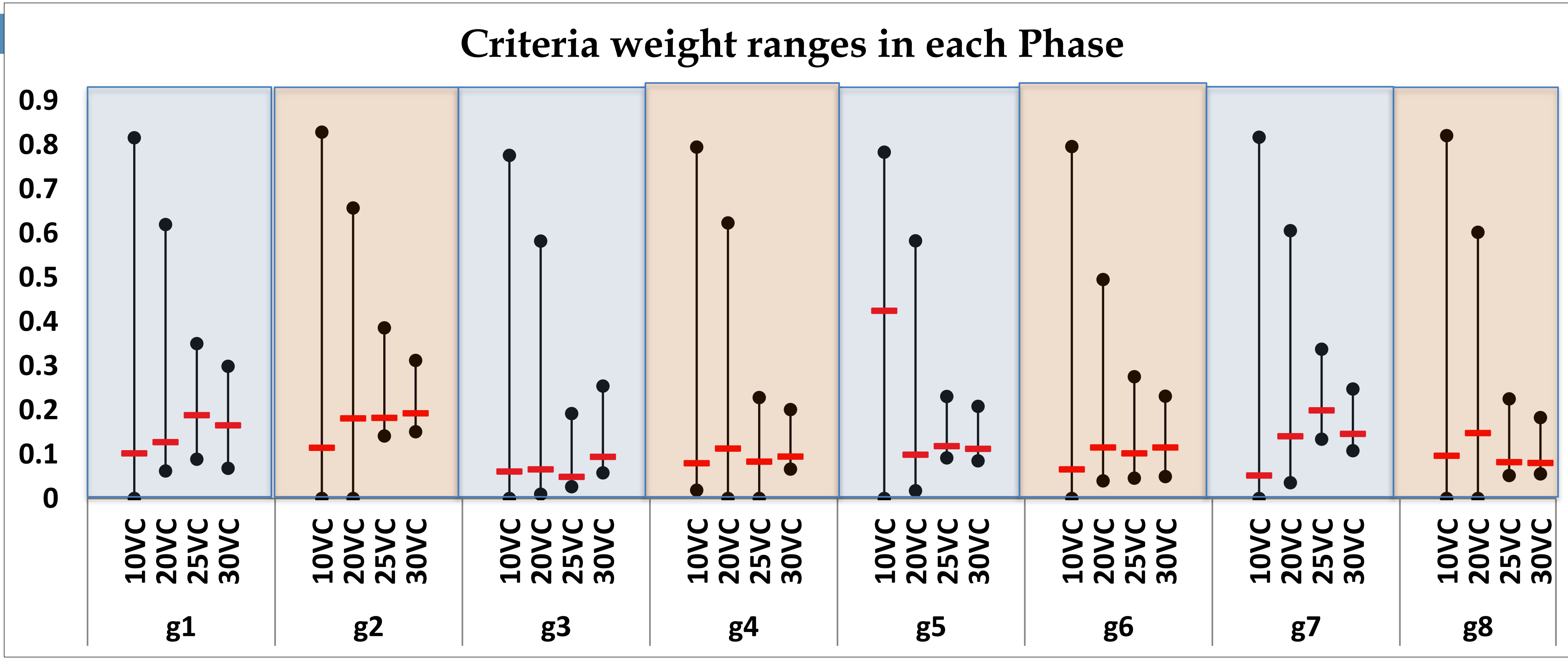
$ARRI = 2.45$

C & D phases – Statistical analysis

	Possible ranks	Rank rate	Possible ranks	Rank rate
Netherlands	1	100.00%	1	100.00%
Finland	2	100.00%	2	100.00%
Sweden	3 - 4 - 5	81.54% - 7.69% - 10.77%	3 - 4	93.85% - 6.15%
France	3 - 4 - 5	18.46% - 67.69% - 13.85%	3 - 4 - 5	6.15% - 78.46% - 15.38%
Denmark	4 - 5 - 6	13.85% - 56.92% - 29.23%	4 - 5 - 6	15,38% - 81.54% - 3.08%
Un. Kingdom	4 - 5 - 6 - 7	10.77% - 18.46% - 67.69% - 3.08%	5 - 6 - 7	3.08% - 80% - 16.92%
Norway	6 - 7	3.08% - 96.92%	6 - 7	16.92% - 83.08%
Austria	8 - 9	18.46% - 81.54%	8 - 9	78.46% - 21.54%
Estonia	8 - 9 - (10)	81.54% - 18.46%	8 - 9	21.54% - 78.46%
Ireland	(9) - 10	100.00%	10	100.00%
Slovenia	11 - 12	21.54% - 78.46%	11 - 12	80.00% - 20.00%
Spain	11 - 12 -13	78.46% - 9.23%	11 - 12 - 13	20% - 43.08% - 36.92%
Belgium	11 - 12 - 13	12.31% - 20% - 67.69%	12 - 13 - 14	36.92% - 47.69% - 15.38%
Portugal	13 - 14 - (15)	67.69% - 32.31%	13 - 14 - 15	15.38% - 49.23% - 35.38%
Germany	14 - 15 - 16 - 17	24.62% - 20.00% - 55.38%	14 - 15 - 16 - 17	35.38% - 43.08% - 6.15% - 15.38%
Slovakia	15 - 16 - 17	20% - 67.69% - 12.31%	15 - 16 - 17	9.23% - 49.23% - 41.54%
Italy	15 - 16 - 17 - 18	55.38% - 12.31% - 26.15% - 6.15%	15 - 16 - 17	12.31% - 44.62% - 43.08%
Poland	(17) - 18 - 19 - 20 - 21	78.46% - 6.15% - 3.08% - 12.31%	18 - 19	89.23% - 10.77%
Czech Rep.	17 - 18 - 19	6.15% - 12.31% - 81.54%	18 - 19	10.77% - 89.23%
Hungary	19 - 20 - 21 - 22	7.69% - 38.46% - 52.31% - 1.54%	20 - 21 - 22	81.54% - 15.38% - 3.08%
Croatia	21 - 22	12.31% - 87.69%	20 - 21 -22	1.54% - 66.15% - 32.31%
Greece	18 - 19 - 20 - 21 - 22	3.08% - 4.62% - 58.46% - 23.08% - 10.77%	20 - 21 - 22	16.92% - 18.46% - 64.62%

25 RCs **30 RCs**

Overall assessment of robustness



Index	20 RCs	25 RCs	30 RCs
ASI	0.936 → +3.74%	*0.971* → +0.93%	*0.980*
ARRI	6.27 → -52.95%	2.95 → -16,95%	2.45

Part VII - Bibliography

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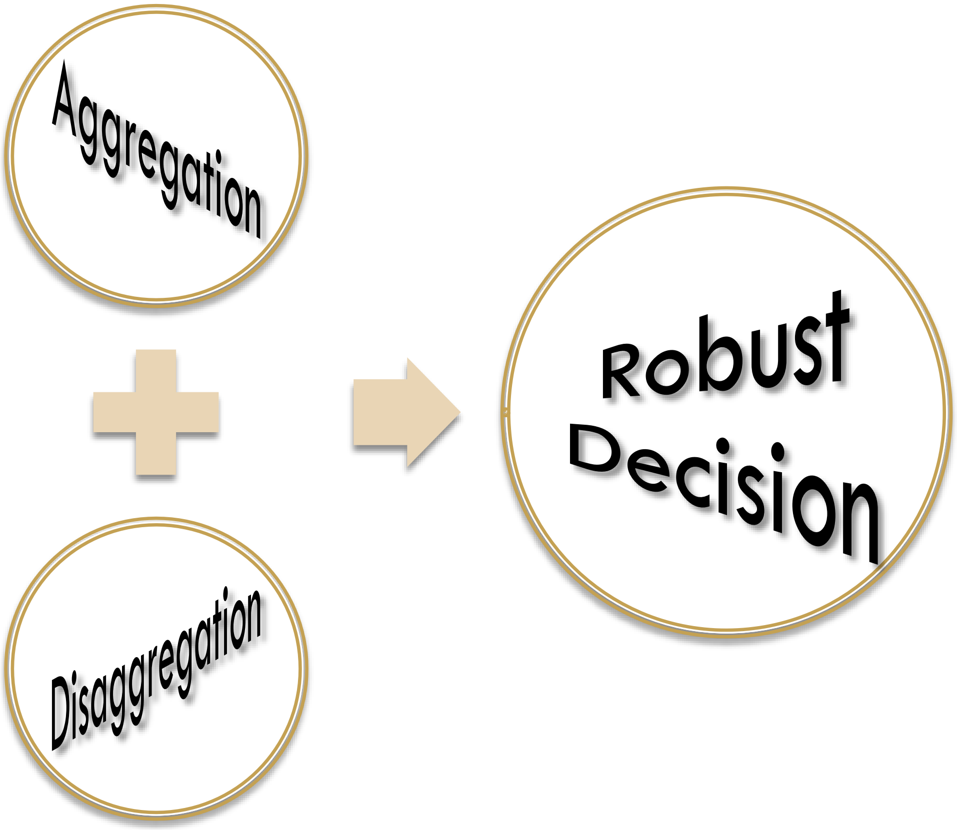

Eleftherios Siskos

National Technical University of Athens
email: lsiskos@epu.ntua.gr