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Multicriteria Satisfaction Analysis

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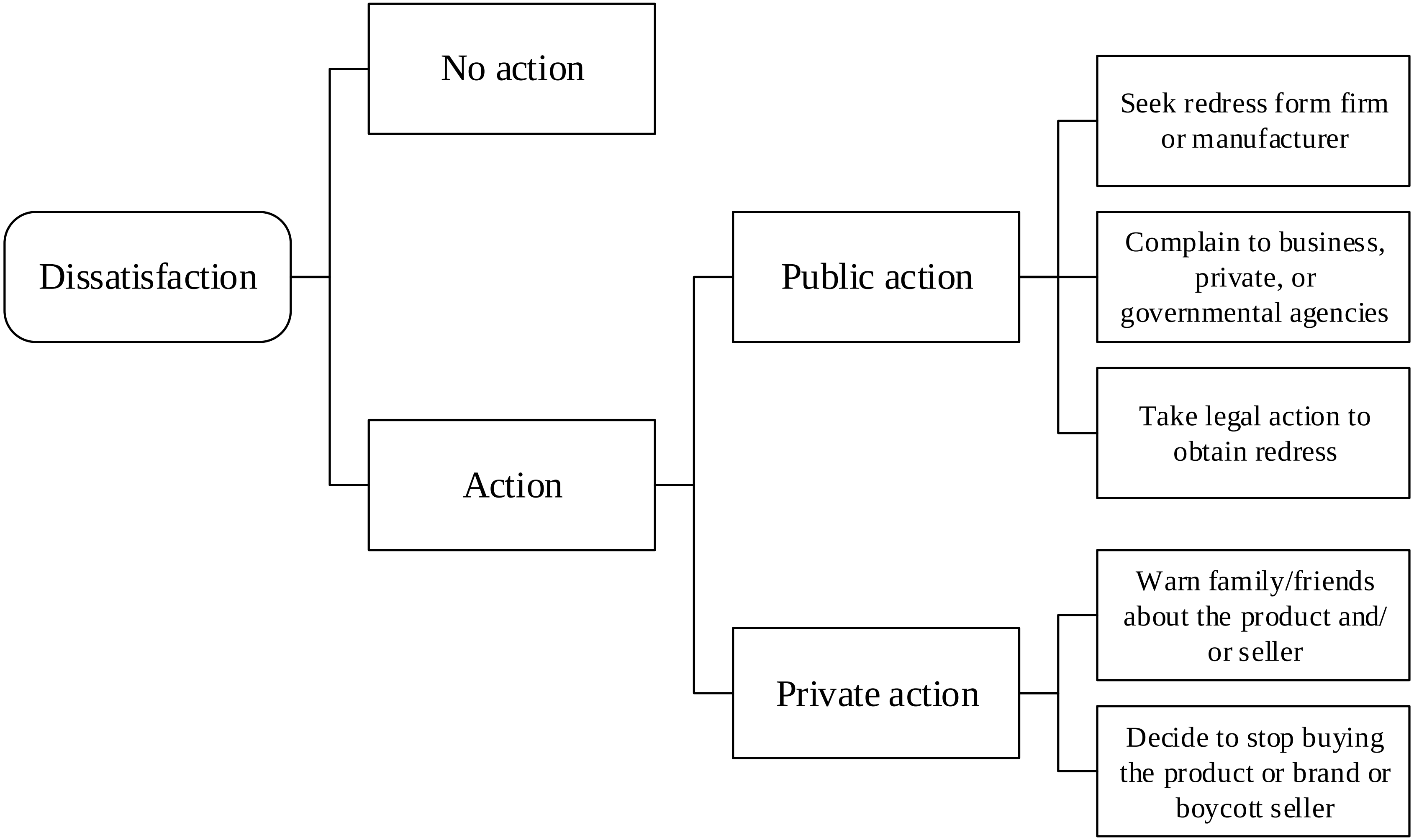
Outline

- Introduction
- MUSA method
  - Basic principles
  - Mathematical development
  - Numerical example
  - Stability analysis
  - Results
- Inconsistencies and other potential problems
- Evaluation of results
- MUSA extensions
  - Strictly increasing functions
  - Multiple criteria levels
  - Alternative post-optimality approaches
  - Robustness
- Concluding remarks

Customer satisfaction measurement

- Importance of customer satisfaction
  - Baseline standard of performance and standard of excellence for any business organization
  - Quality management and quality assurance standards
  - Availability of different forms of customer satisfaction info (e.g., social media, online surveys, rating systems)
- Customer satisfaction measurement
  - Measurement offers an immediate, meaningful, and objective feedback
  - Measurement indicates what should be improved, and the ways through which this improvement could be achieved
  - Measurement provides a sense of achievement and accomplishment
  - Need to translate satisfaction data into a number of measurable parameters

Customer dissatisfaction



Motivation

- Basic characteristics of the customer satisfaction measurement problem
  - Customer satisfaction depends on a set of product/service quality characteristics
  - Satisfaction information is directly available by customers (e.g., surveys)
  - Usually available info has an ordinal form
- Standard approaches
  - Regression-type models
  - Statistical or other data analysis tools
  - No (or limited) MCDA methods

Example of customer satisfaction data

Hilton hotel service experience

Quitter ce sondage

1.

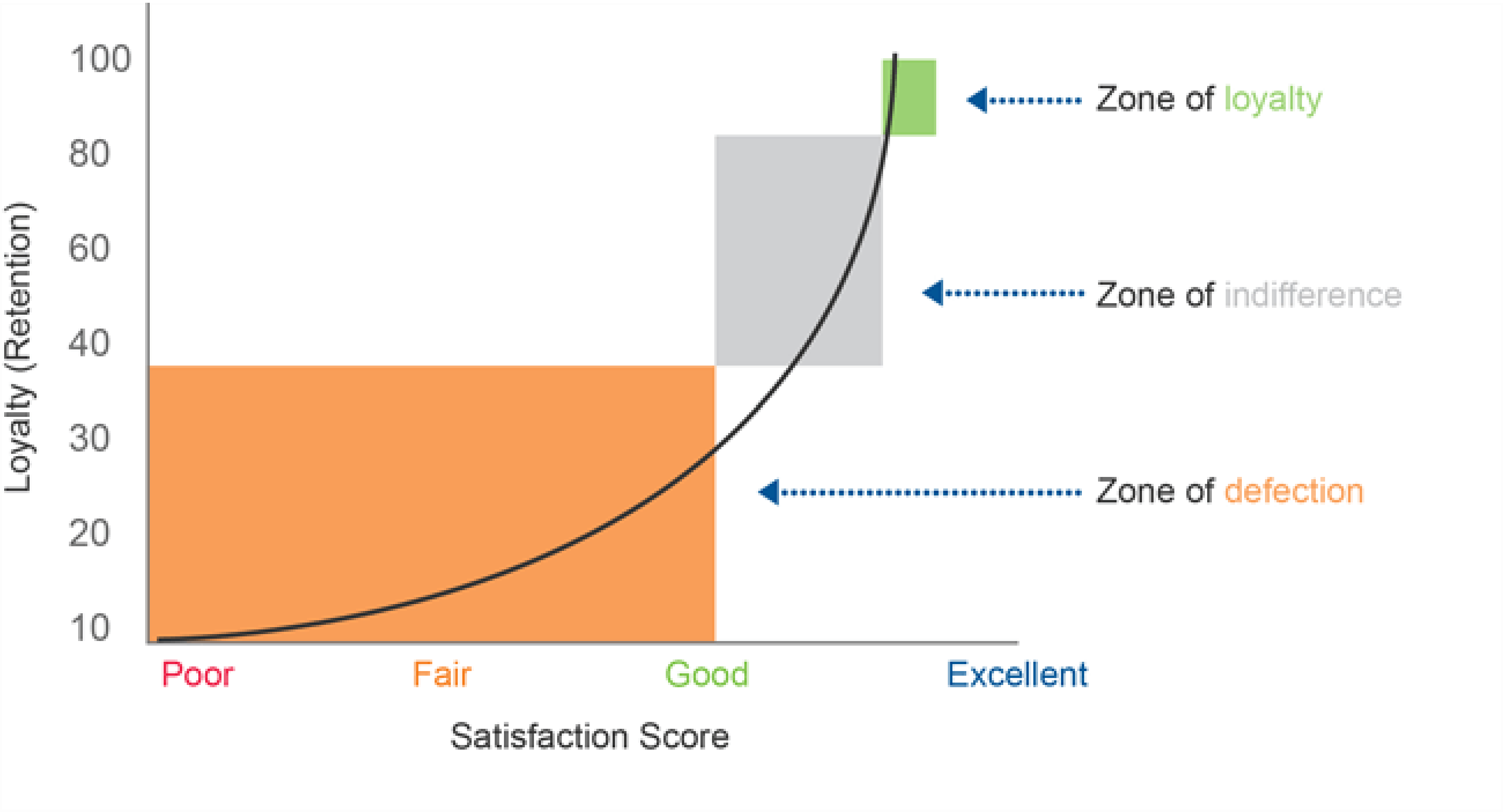
1. How would you rate the following services at the Hilton hotel?

	Very dissatisfied	Dissatisfied	Neutral	Satisfied	Very satisfied
Customer service	○	○	○	○	○
Restaurant service	○	○	○	○	○
Bar service	○	○	○	○	○
Room service	○	○	○	○	○
Housekeeping	○	○	○	○	○
Overall service delivery	○	○	○	○	○

‘Typical’ approach in statistical methods

- Transform ordinal to cardinal scale
  - Very dissatisfied → 1
  - Dissatisfied → 2
  - Neutral → 3
  - Satisfied → 4
  - Very satisfied → 5
- …but such transformation
  - Seems arbitrary
  - It is only one of an infinite number of *permissible* transformations (Stevens, 1946 on the theory of scales on measurement)
  - Assumes a ‘linear’ customer satisfaction behavior

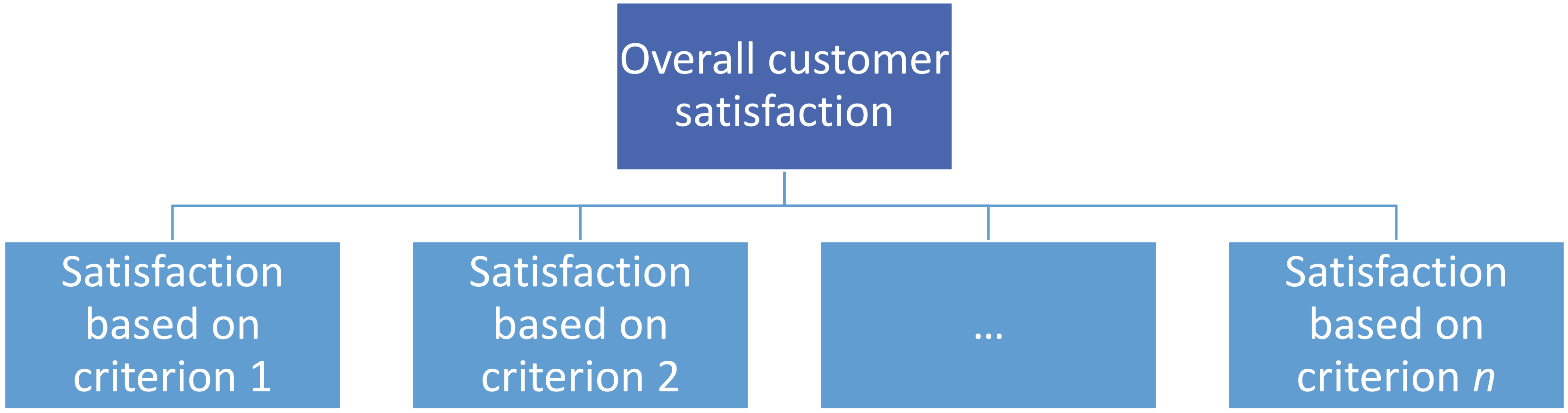
Customer satisfaction vs customer loyalty



The MUSA method

- It is a consumer-based method, since it requires input survey data using a questionnaire of a certain type
- It assumes that customer’s global satisfaction is based on a set of criteria representing product/service quality characteristics
- It is an ordinal regression-based approach used for the assessment of a set of collective satisfaction functions in such a way that the global satisfaction criterion becomes as consistent as possible with customers’ judgements

Satisfaction criteria hierarchy



Ordinal regression approach

- The main object of the MUSA method is the aggregation of individual judgements into a collective value function
- MUSA is a preference disaggregation method used for the assessment of global and partial satisfaction functions Y^* and X_i^* respectively, given customers' judgements Y and X_i
- Ordinal regression equation:

$$Y^* = \sum_{i=1}^{n} b_i X_i^* \text{ with } \sum_{i=1}^{n} b_i = 1$$

where Y^* and X_i^* are the global and marginal value (satisfaction) functions and b_i is the weighting factor of criterion i

Value (satisfaction) functions

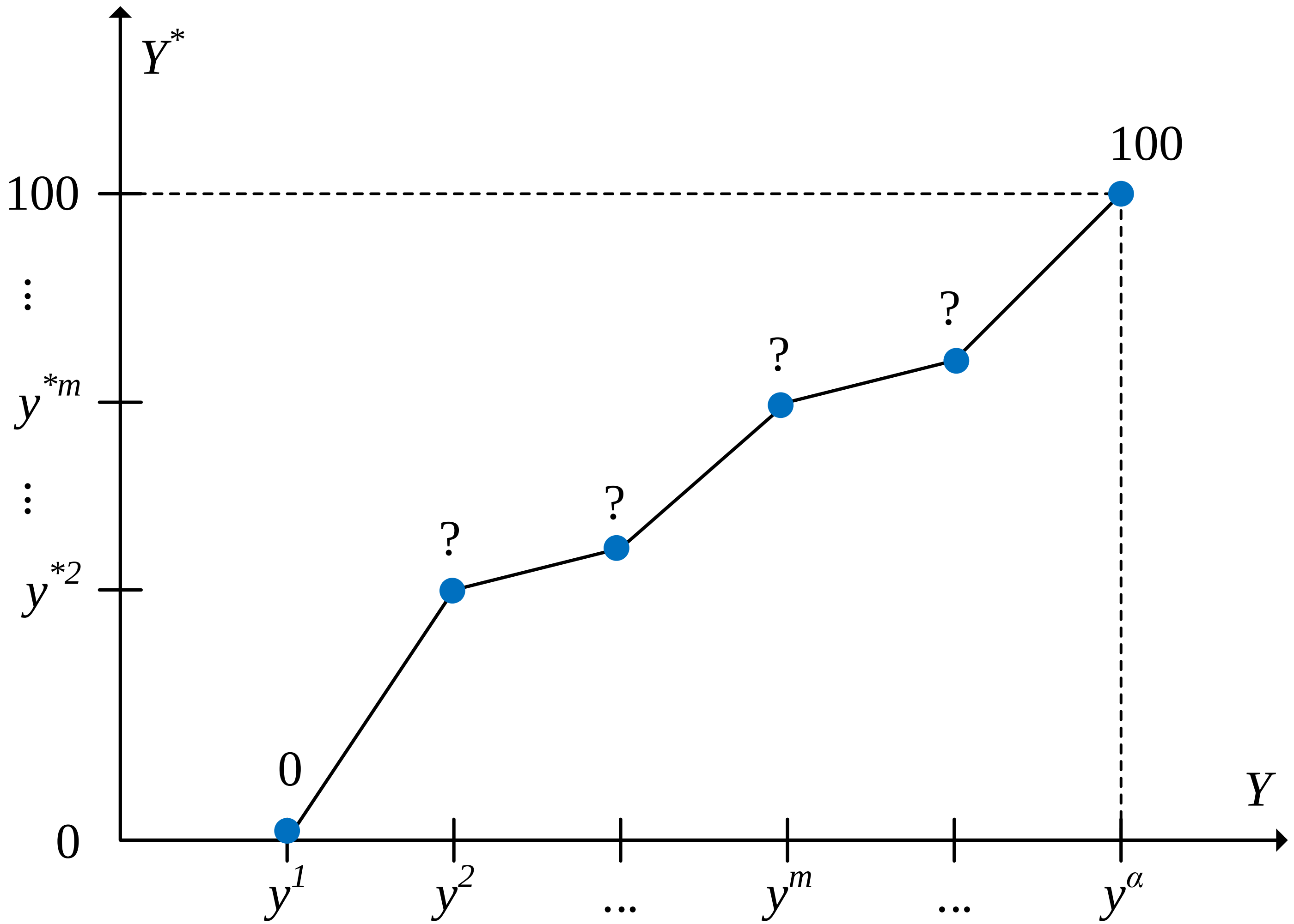
- Value functions Y^* and X_i^* are normalized in [0,100]

$$\begin{cases} y^{*1} = 0, y^{*\alpha} = 100 \\ x_i^{*1} = 0, x_i^{*\alpha_i} = 100 \; i = 1,2, \dots n \end{cases}$$

- Preference conditions

$$\begin{cases} y^{*m} \leq y^{*m+1} \\ x_i^{*k} \leq x_i^{*k+1} \end{cases} \Leftrightarrow \begin{cases} y^{m} \preceq y^{m+1}, m = 1,2, \dots, \alpha - 1 \\ x_i^{k} \preceq x_i^{k+1}, k = 1,2, \dots, \alpha_i - 1 \end{cases}$$

Value (satisfaction) functions



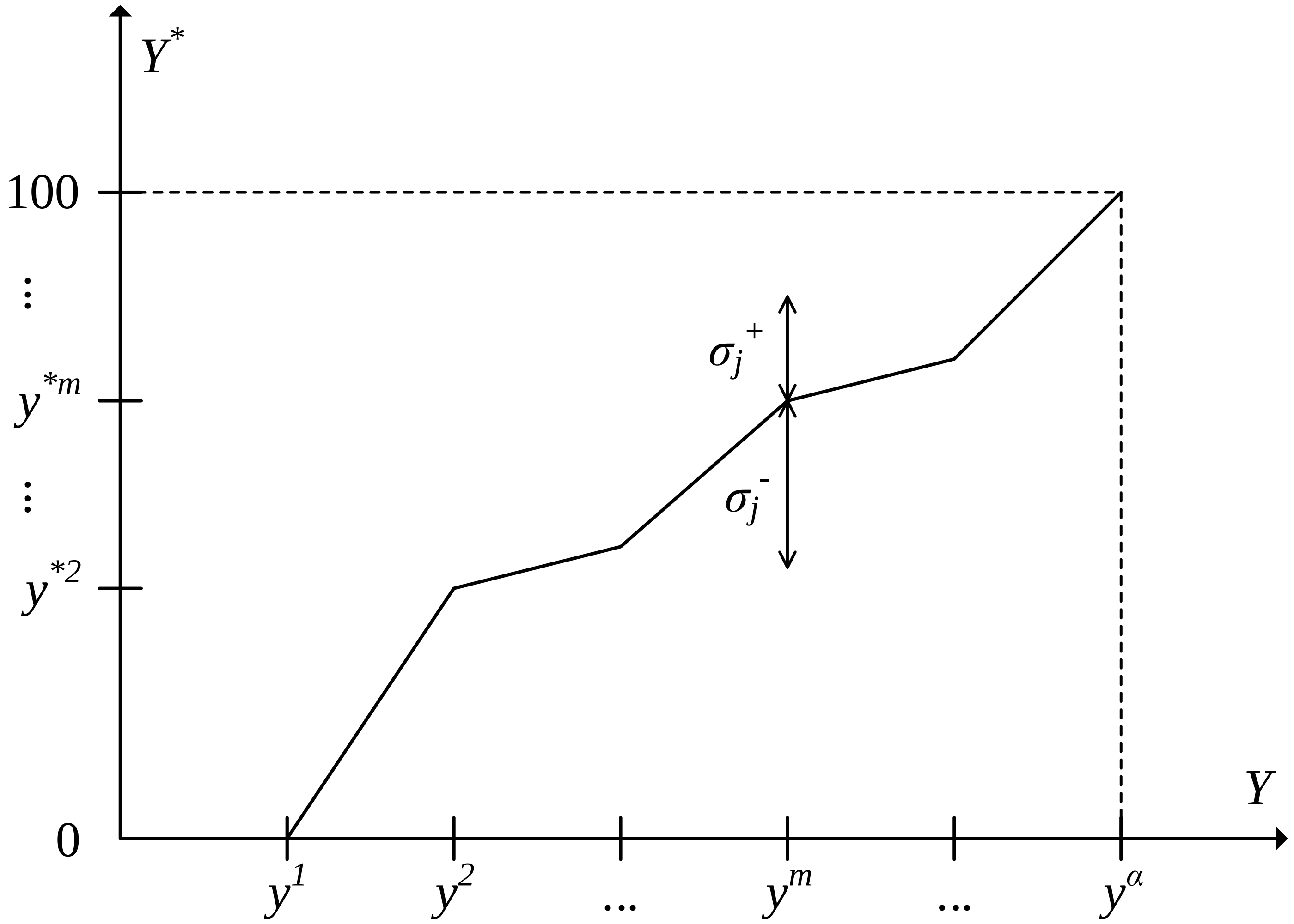
Model development

- Introduce a double error variable:

$$\tilde{Y}^* = \sum_{i=1}^{n} b_i X_i^* + \sigma^+ + \sigma^-$$

  where $\tilde{Y}^*$ is the estimation of the overall value function and σ^+, σ^- are the overestimation and the underestimation errors
- Error variables are assessed for each customer separately
- Similar to goal programming

Value (satisfaction) functions with error variables



LP formulation

- Minimize the sum of errors

under the constraints:

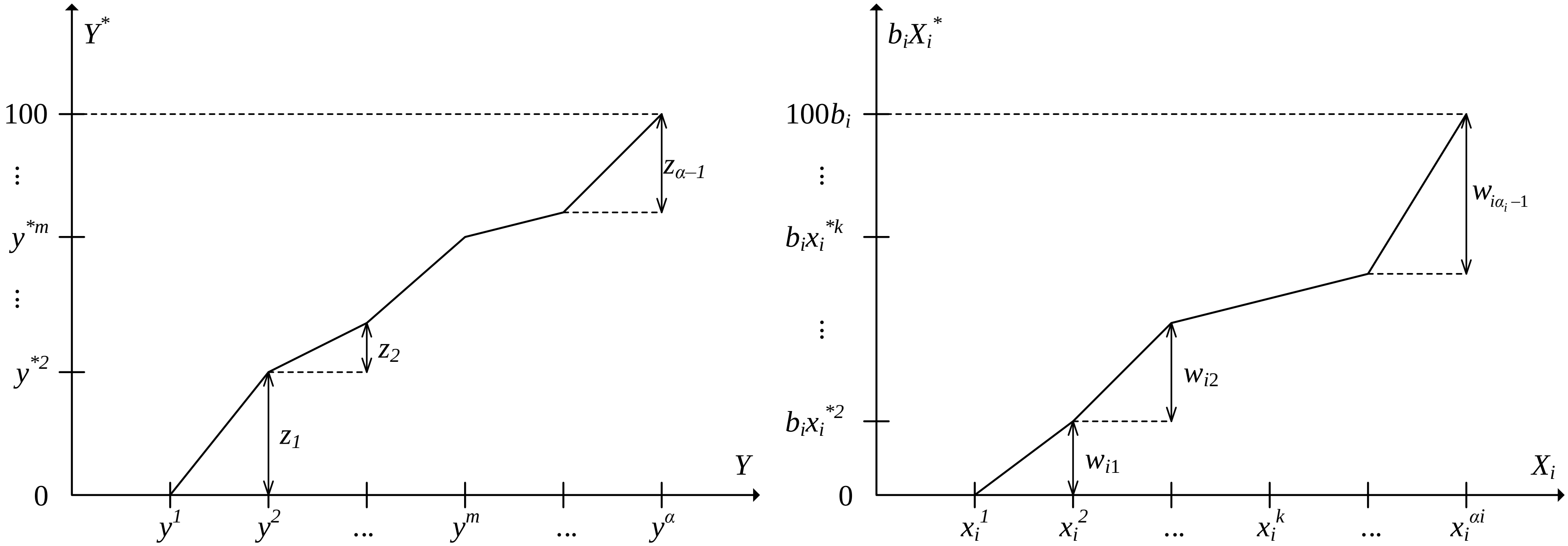
- ordinal regression equation for each customer,
- normalization constraints for Y^* and X_i^* in [0, 100],
- monotonicity constraints for Y^* and X_i^*

Transformations

- Transformations: successive steps of the value functions Y^* and X_i^*
- Benefits
  - Remove the monotonicity constraints
  - Formulate a linear model

$$\begin{cases} z_m = y^{*m+1} - y^{*m}, m = 1,2,\ldots,\alpha - 1 \\ w_{ik} = x_i^{*k+1} - x_i^{*k}, k = 1,2,\ldots,\alpha_i - 1, i = 1,2,\ldots,n \end{cases}$$

Transformations



Basic LP

$$[min]F = \sum_{j=1}^{M} \sigma_j^+ + \sigma_j^-$$

under the constrains

$$\sum_{i=1}^{n} \sum_{k=1}^{x_i^j - 1} w_{ik} - \sum_{m=1}^{y^j - 1} z_m - \sigma_j^+ + \sigma_j^- = 0$$

$$\sum_{m=1}^{\alpha - 1} z_m = 100$$

$$\sum_{i=1}^{n} \sum_{k=1}^{\alpha_i - 1} w_{ik} = 100$$

$$z_m, w_{ik}, \sigma_j^+, \sigma_j^- \geq 0 \; \forall m, i, k, j$$

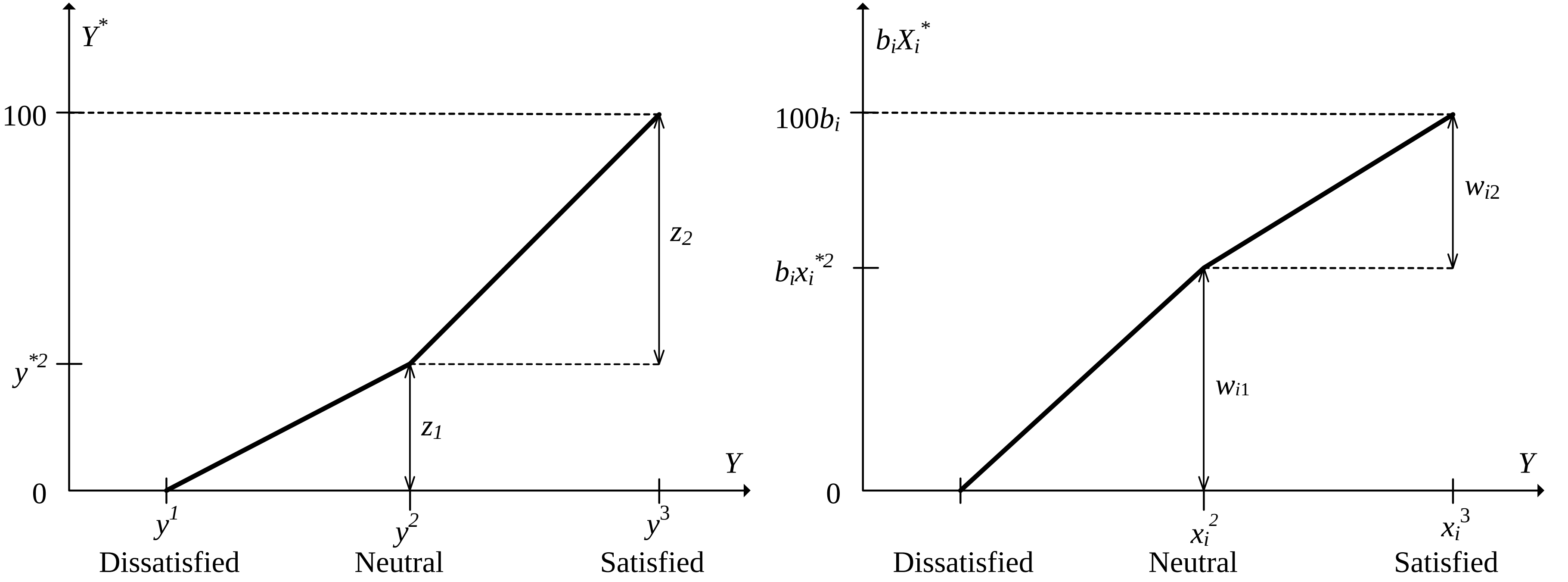
Numerical example

- Consider a simple case of customer satisfaction measurement:
  - 3 satisfaction criteria (product, purchase process, and service).
  - Sample of 20 customers’ judgements
  - 3point ordinal satisfaction scale (i.e., dissatisfied, neutral, satisfied).

Numerical example (data)

Overall satisfaction	Satisfaction from criterion 1	Satisfaction from criterion 2	Satisfaction from criterion 3
Neutral	Satisfied	Neutral	Dissatisfied
Dissatisfied	Dissatisfied	Dissatisfied	Dissatisfied
Satisfied	Satisfied	Satisfied	Satisfied
Neutral	Satisfied	Dissatisfied	Neutral
Dissatisfied	Dissatisfied	Dissatisfied	Dissatisfied
Satisfied	Satisfied	Satisfied	Satisfied
Neutral	Satisfied	Dissatisfied	Satisfied
Neutral	Satisfied	Dissatisfied	Satisfied
Neutral	Neutral	Neutral	Neutral
Dissatisfied	Dissatisfied	Dissatisfied	Dissatisfied
Neutral	Neutral	Satisfied	Dissatisfied
Dissatisfied	Dissatisfied	Dissatisfied	Dissatisfied
Satisfied	Satisfied	Satisfied	Satisfied
Neutral	Neutral	Satisfied	Dissatisfied
Dissatisfied	Dissatisfied	Dissatisfied	Dissatisfied
Satisfied	Satisfied	Satisfied	Neutral
Satisfied	Satisfied	Satisfied	Satisfied
Satisfied	Satisfied	Satisfied	Neutral
Neutral	Neutral	Neutral	Neutral
Dissatisfied	Neutral	Dissatisfied	Dissatisfied

Numerical example (variables)



Numerical example (modeling)

- Objective function: $\sum_{j=1}^{20} \sigma_j^+ + \sigma_j^-$
- Writing ordinal regression equations, e.g. for customer 1:
  - Criterion 1 (satisfied): $w_{11} + w_{12}$
  - Criterion 2 (neutral): w_{21}
  - Criterion 3 (dissatisfied): 0
  - Overall satisfaction (neutral): z_1
  - Thus: $w_{11} + w_{12} + w_{21} - z_1 - \sigma_1^+ + \sigma_1^- = 0$
- Normalization constraints:
  - Overall value function: $z_1 + z_2 = 100$
  - Marginal value functions:
    $w_{11} + w_{12} + w_{21} + w_{22} + w_{31} + w_{32} = 100$

Numerical example (basic LP)

w_{11}	w_{12}	w_{21}	w_{22}	w_{31}	w_{32}	z_1	z_2	σ_1^+	σ_1^-	σ_2^+	σ_2^-	σ_3^+	σ_3^-	σ_4^+	σ_4^-	σ_5^+	σ_5^-	σ_6^+	σ_6^-	σ_7^+	σ_7^-	σ_8^+	σ_8^-	σ_9^+	σ_9^-	σ_{10}^+	σ_{10}^-	σ_{11}^+	σ_{11}^-	σ_{12}^+	σ_{12}^-	σ_{13}^+	σ_{13}^-	σ_{14}^+	σ_{14}^-	σ_{15}^+	σ_{15}^-	σ_{16}^+	σ_{16}^-	σ_{17}^+	σ_{17}^-	σ_{18}^+	σ_{18}^-	σ_{19}^+	σ_{19}^-	σ_{20}^+	σ_{20}^-		β^*
1	1	1				-1		-1	1																																							=	0
										-1	1																																					=	0
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Numerical example (solution)

Variable	Value
w_{11}	0
w_{12}	25
w_{21}	25
w_{22}	25
w_{31}	25
w_{32}	0
z_1	50
z_2	50
F^*	0

Stability analysis

- Problem of multiple or near optimal solutions
- Stability analysis is considered as a post-optimality problem.
- This solution is calculated by *n* LPs (equal to the number of criteria), which maximize the weight of each criterion:

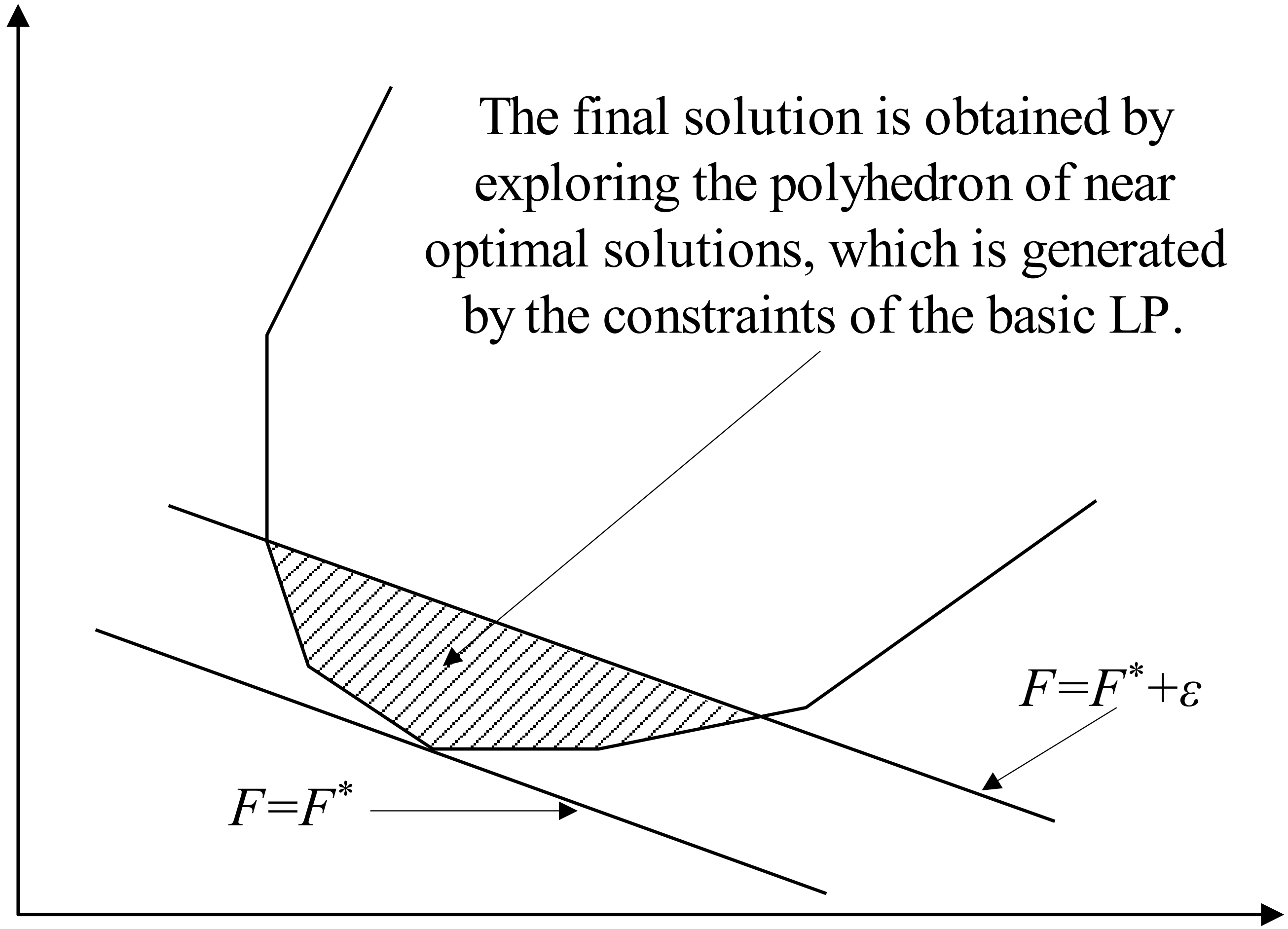
$$[max]F' = \sum_{k=1}^{\alpha_i - 1} w_{ik}$$

Subject to

$$F \leq F^* + \varepsilon$$

All the constrains of the basic LP

Stability analysis



Stability analysis

- The average of these optimal solutions may be considered as the final solution of the problem.
- In case of instability
  - A large variation of the provided solutions appears in the post-optimality analysis, and
  - The final average solution is less representative

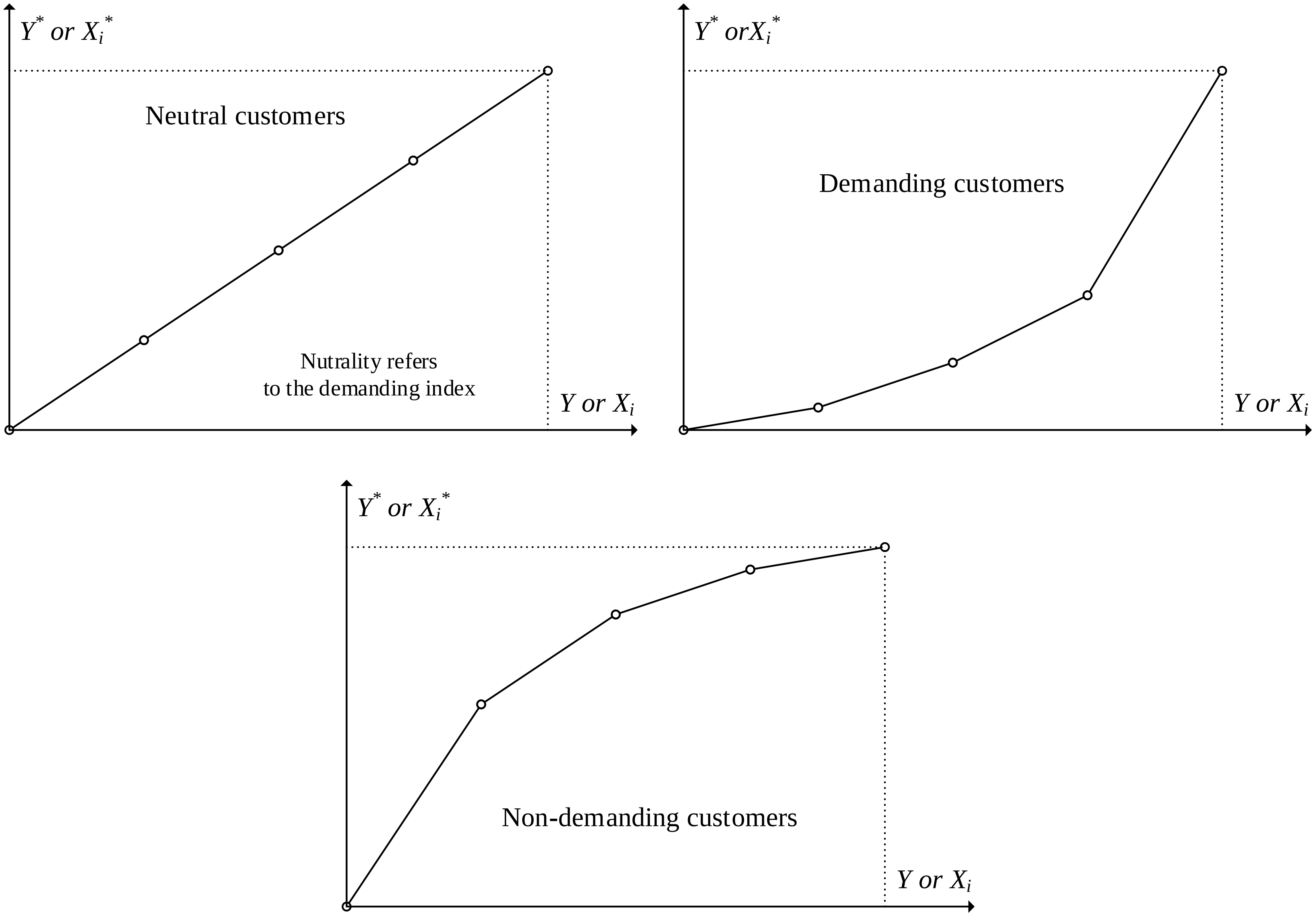
Numerical example (post-optimality analysis

	w_{11}	w_{12}	w_{21}	w_{22}	w_{31}	w_{32}	z_1	z_2
$\max b_1$	10.00	22.50	22.50	22.50	22.50	0.00	55.00	45.00
$\max b_2$	0.00	23.75	23.75	28.75	23.75	0.00	47.50	52.50
$\max b_3$	0.00	20.00	20.00	30.00	30.00	0.00	50.00	50.00
Average	3.33	22.08	22.08	27.08	25.42	0.00	50.83	49.17

Results (value functions)

- These functions show the real value (in a normalized interval 0-100) that customers give for each level of the global or partial ordinal satisfaction scale
- The global and partial value functions Y^* and X_i^* respectively, are mentioned as additive and marginal value or utility functions, and their properties are determined in the context of multicriteria analysis
- They are monotonic, non-decreasing, discrete (piecewise linear) functions
- The form of these curves indicate if customers are demanding

Results (value functions)



Results (criteria weights)

- The criteria weights represent the relative importance of the assessed satisfaction criteria
- The decision whether a satisfaction dimension is considered important by the customers is also based on the number of assessed criteria
- The properties of the weights are determined in the context of multicriteria analysis
- The weights are value trade-offs among the criteria

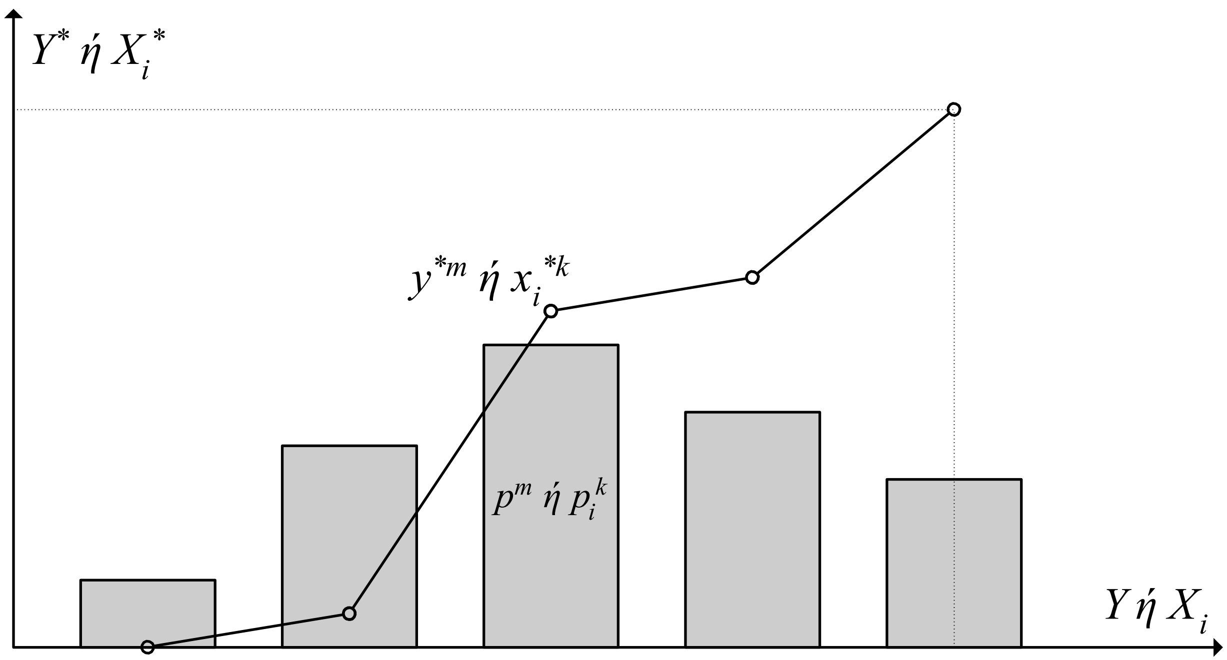
Results (average satisfaction indices

- The assessment of a performance norm may be very useful in customer satisfaction analysis
- The average global and partial satisfaction indices are used for this purpose, and can be assessed according to the following equations:

$$\begin{cases} S = \dfrac{1}{100} \sum_{m=1}^{\alpha} p^m \, y^{*m} \\ S_i = \dfrac{1}{100} \sum_{k=1}^{\alpha_i - 1} p_i^k x_i^{*k} \text{ for } i = 1,2,\ldots,n \end{cases}$$

Results (average satisfaction indices

- These indices are normalized (0-100%)
- The average satisfaction indices are basically the mean value of the global and partial value functions



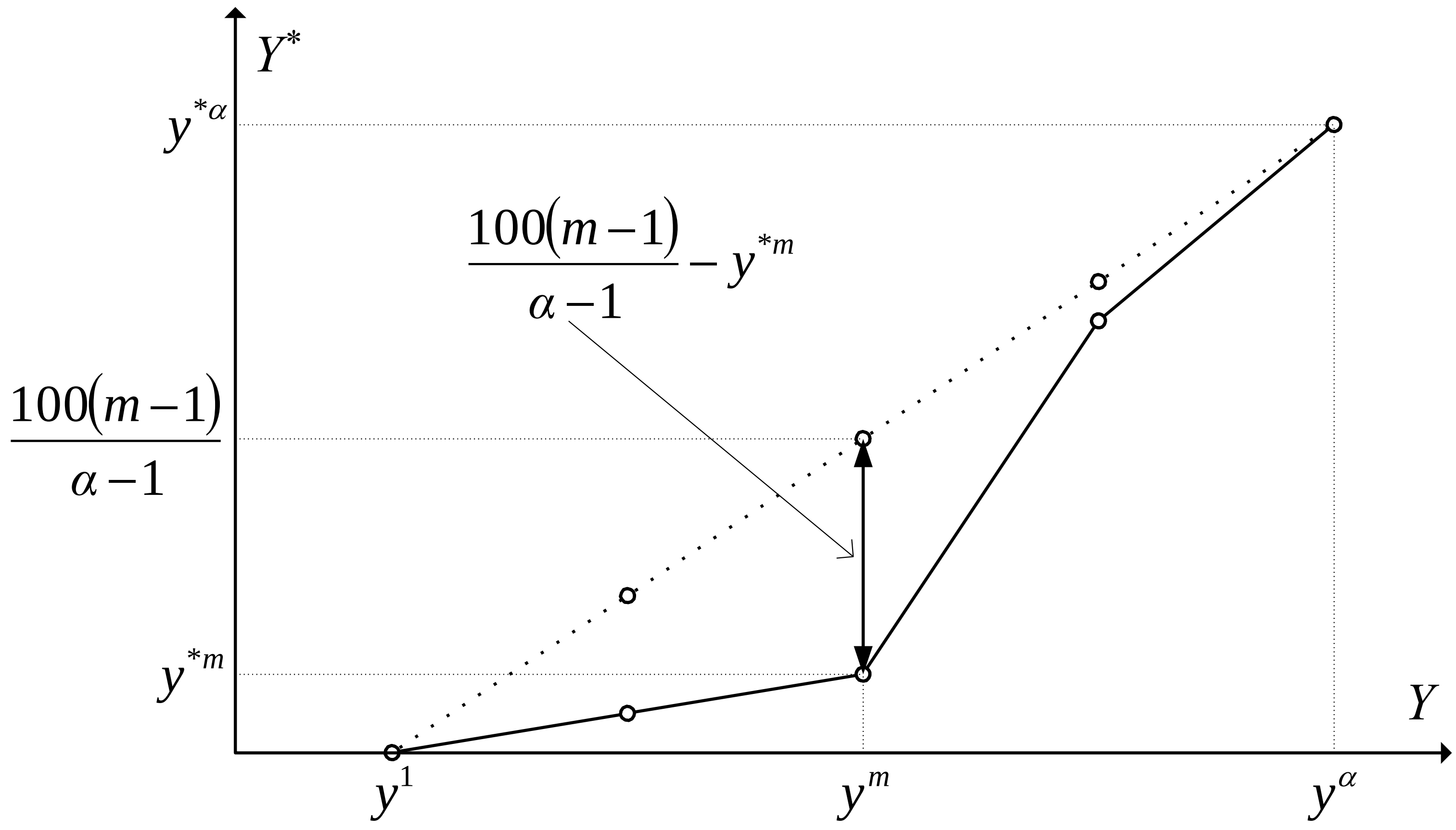
Results (average demanding indices

- The average global and partial demanding customer indices are assessed according to the following equations:

$$\begin{cases} D = \dfrac{\sum_{m=1}^{\alpha-1}\left(\dfrac{100(m-1)}{\alpha-1} - y^{*m}\right)}{100\sum_{m=1}^{\alpha-1}\dfrac{m-1}{\alpha-1}} \text{ for } \alpha > 2 \\ D_i = \dfrac{\sum_{k=1}^{\alpha_i-1}\left(\dfrac{100(k-1)}{\alpha_i-1} - x_i^{*k}\right)}{100\sum_{k=1}^{\alpha_i-1}\dfrac{k-1}{\alpha_i-1}} \text{ for } \alpha_i > 2 \text{ and } i = 1,2,\dots,n \end{cases}$$

- The shape of global and partial satisfaction functions indicates customers' demanding level
- These indices represent the average deviation of the estimated value functions from a "normal" (linear) function

Results (average demanding indices



Results (average demanding indices

- The average demanding indices are normalized in [-1, 1] and the following possible cases hold:
  - Neutral customers ($D = 0$ or $D_i = 0$)
  - Demanding customers ($D = 1$ or $D_i = 1$)
  - Non-demanding customers ($D = -1$ or $D_i = -1$)
- Demanding indices can be used for customer behavior analysis, and they can also indicate the extent of company's improvement efforts: the higher the value of the demanding index, the more the satisfaction level should be improved in order to fulfil customers' expectations.

Results (average improvement indices)

- The output of improvement efforts depends on the importance of the satisfaction criteria and their contribution to dissatisfaction
- The average improvement indices are assessed according to the following equation:

$$I_i = b_i(1 - S_i) \text{ for } i = 1,2,\dots,n$$

- These indices are normalized in [0, 1] and it can be proved that:
- $\begin{cases} I_i = 1 \Leftrightarrow b_i = 1 \wedge S_i = 0 \\ I_i = 0 \Leftrightarrow b_i = 0 \vee S_i = 1 \end{cases} \text{ for } i = 1,2,\dots,n$
- These indices can show the improvement margins on a specific criterion

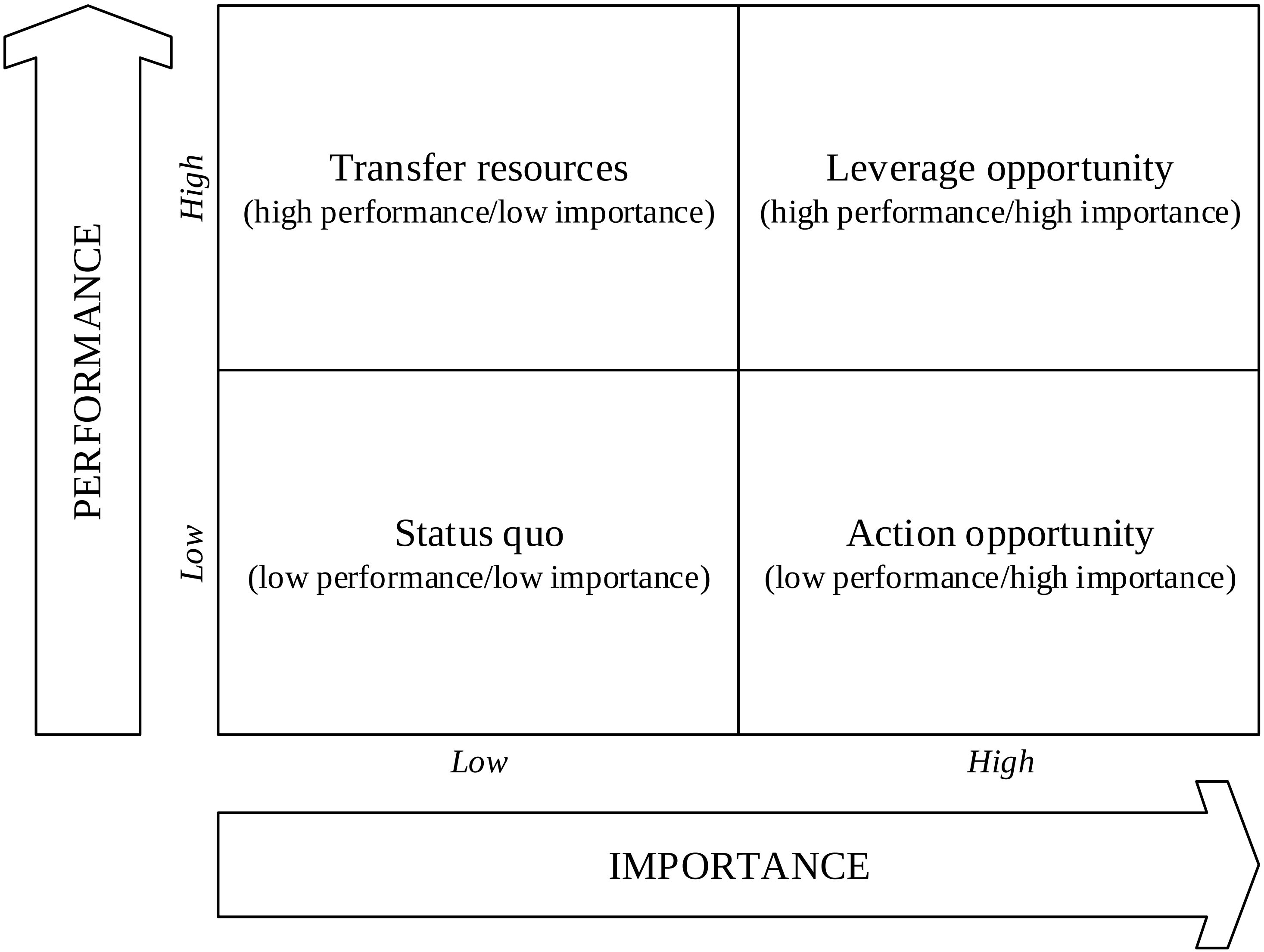
Results (action diagrams)

- Combining weights and satisfaction indices, a series of “Performance/Importance” diagrams can be developed
- These diagrams are also mentioned as action, decision, and strategic or perceptual maps; they are very similar to SWOT analysis
- Each of these maps is divided into quadrants according to performance (high/low), and importance (high/low), that may be used to classify actions.

Results (action diagrams)

- **Status quo**: generally, no action is required
- **Leverage opportunity**: these areas can be used as advantage against competition
- **Transfer resources**: company's resources may be better used elsewhere
- **Action opportunity**: these are the criteria/ subcriteria that need attention

Results (action diagrams)



Results (action diagrams)

- There are 2 types of diagrams:
  - **Raw diagram**: it uses the weights and the satisfaction indices as they are calculated by the MUSA method
  - **Relative diagram**: the cut-off level for axes is recalculated as the centroid of all points in the diagram; this type of diagram is very useful if points are concentrated in a small area
- The relative diagrams use the normalized variables b_i' and S_i' which can overcome:
  - The assessment problem of the cut-off level for axes, and
  - The low-variation problem for the average satisfaction indices (high competitive market case).

Results (action diagrams)

Action diagram	Axes	Variables	Interval	Cut-off level for axes
Raw	Importance	b_i	[0, 1]	$1/n$
	Performance	S_i	[0, 1]	0.5
Relative	Importance	$b_i' = \frac{b_i - \bar{b}}{\sqrt{\left(b_i - \bar{b}\right)^2}}$	[-1, 1]	0
	Performance	$S_i' = \frac{S_i - \bar{S}}{\sqrt{\left(S_i - \bar{S}\right)^2}}$	[-1, 1]	0

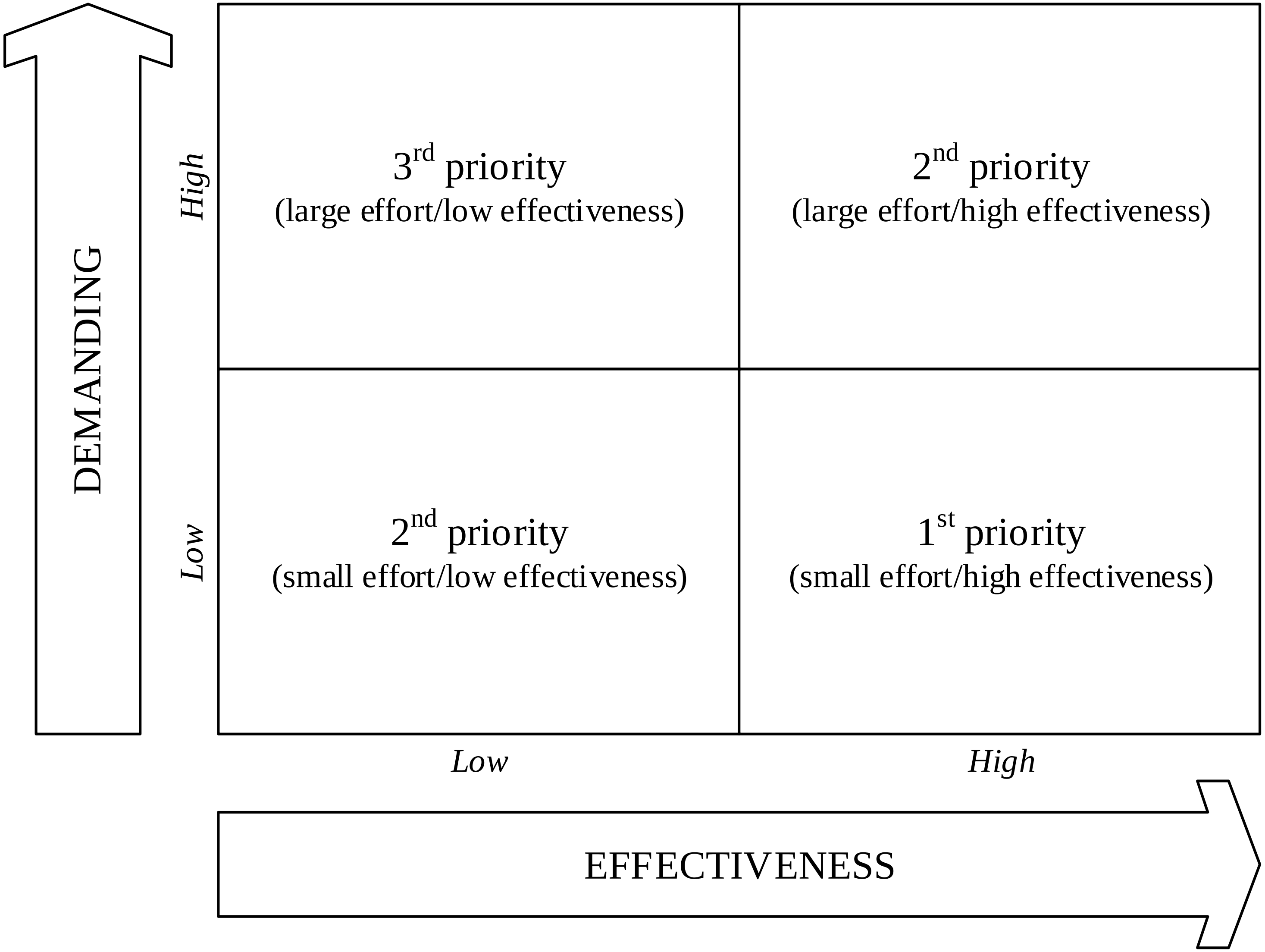
Results (improvement diagrams)

- The action diagrams can indicate which satisfaction criteria should be improved, but they cannot determine the output or the extent of improvement efforts
- For this reason, combining improvement and demanding indices, a series of improvement diagrams can be developed
- Each of these maps is divided into quadrants according to demanding (high/low), and effectiveness (high/low), that may be used to rank improvement priorities

Results (improvement diagrams)

- **1st priority**: this area can indicate improvement actions since these dimensions are highly effective and customers are no demanding
- **2nd priority**: it includes satisfaction dimensions which have either a low demanding index or a high improvement index
- **3rd priority**: it refers to satisfaction dimensions that have small improvement margin and need substantial effort

Results (improvement diagrams)



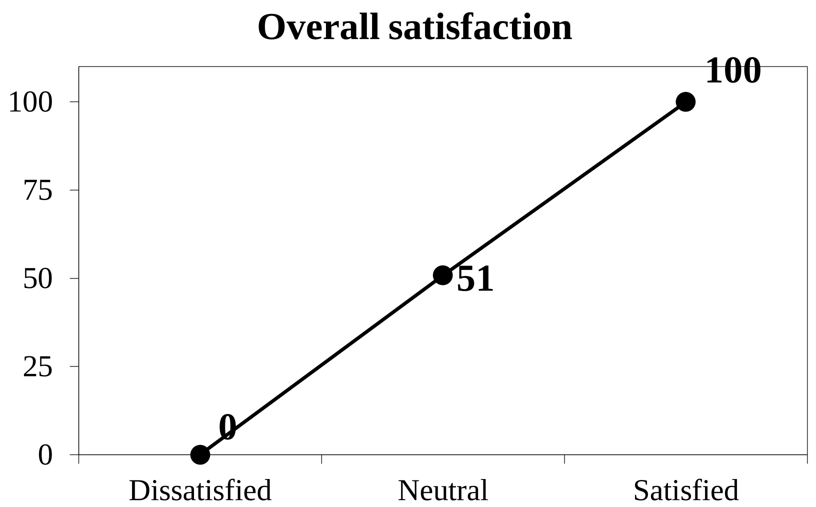
Results (improvement diagrams)

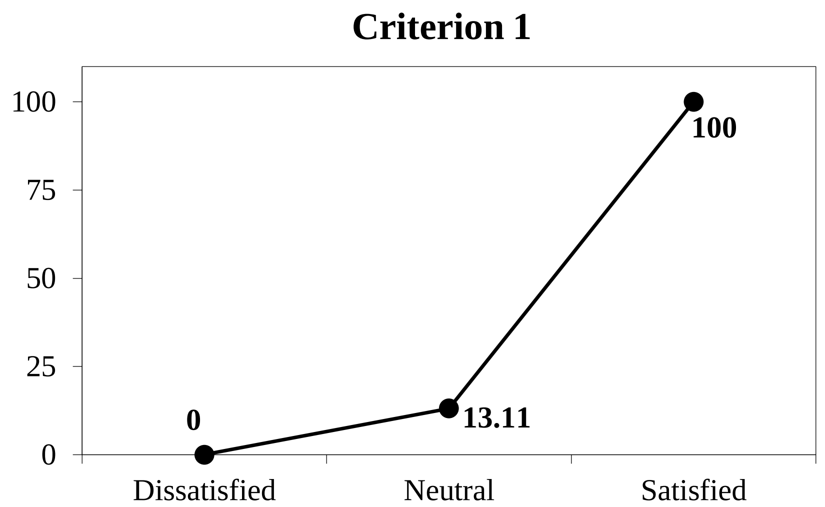
Improvement diagram	Axes	Variables	Interval	Cut-off level for axes
Raw	Effectiveness	I_i	[0, 1]	0.5
	Demanding	D_i	[-1, 1]	0
Relative	Effectiveness	$I_i' = \frac{I_i - \bar{I}}{\sqrt{\left(I_i - \bar{I}\right)^2}}$	[-1, 1]	0
	Demanding	D_i'	[-1, 1]	0

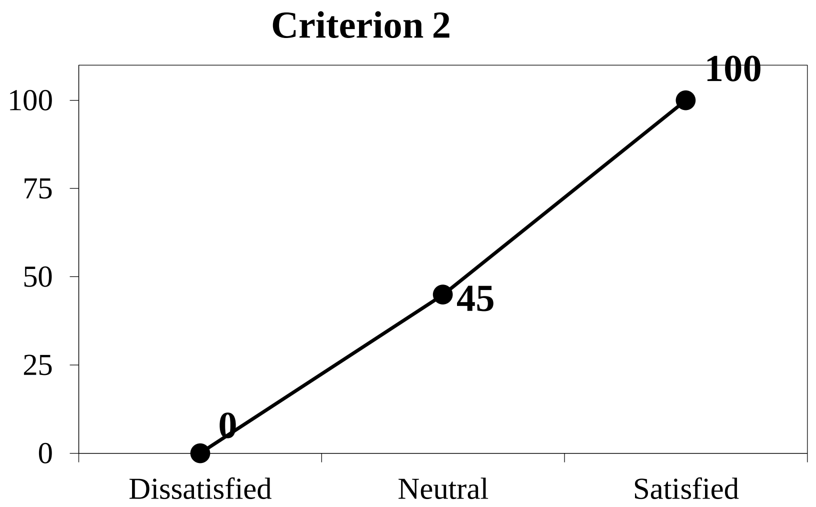
Numerical example (results)

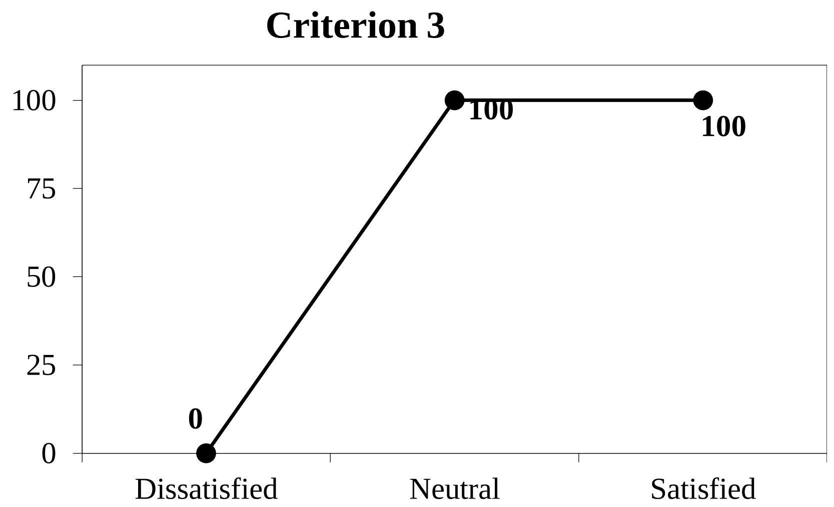
Criterion	Weight	Average satisfaction index	Average demanding index
Criterion 1	0.2542	0.5328	0.74
Criterion 2	0.4917	0.4674	0.10
Criterion 3	0.2542	0.5500	-1.00
Overall satisfaction		**0.5033**	**-0.02**

Numerical example (results)

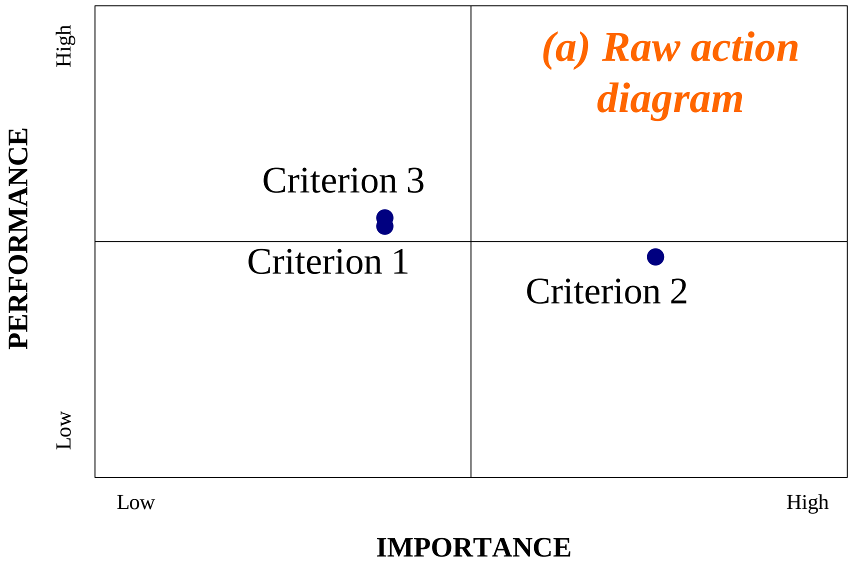


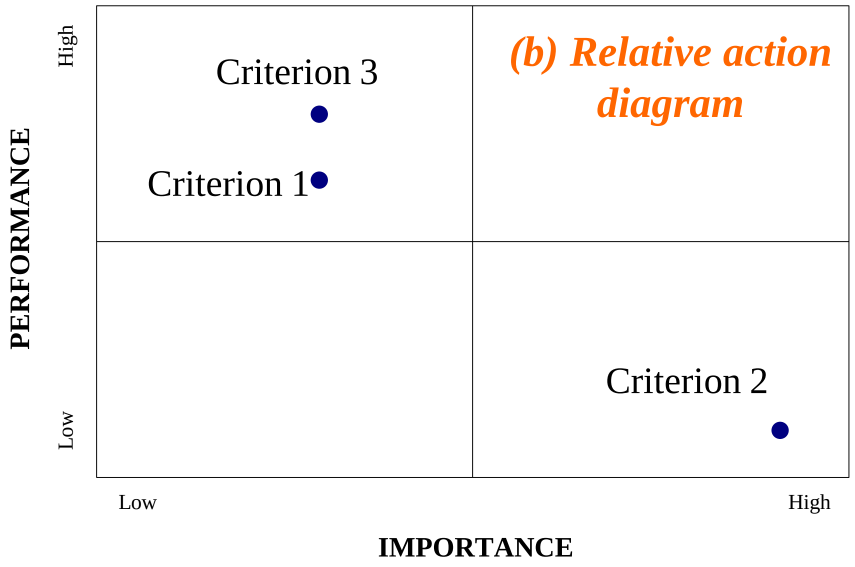


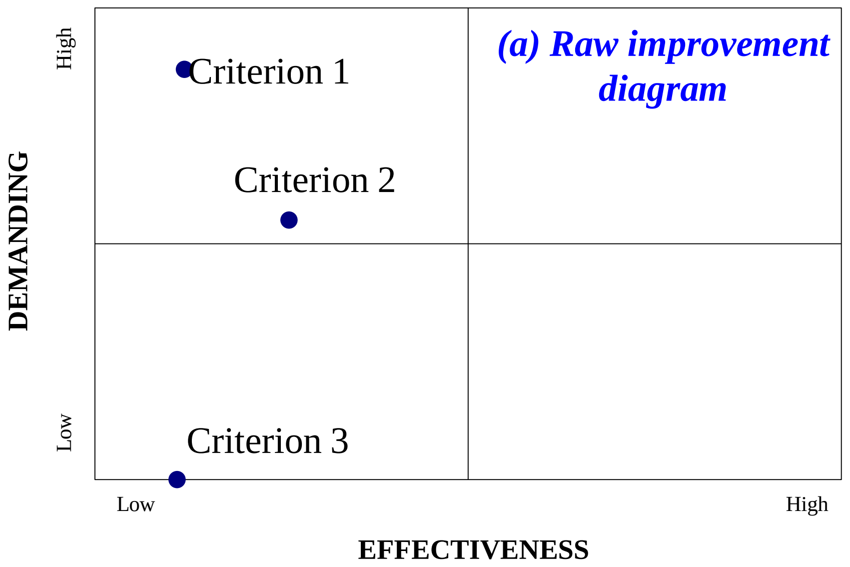


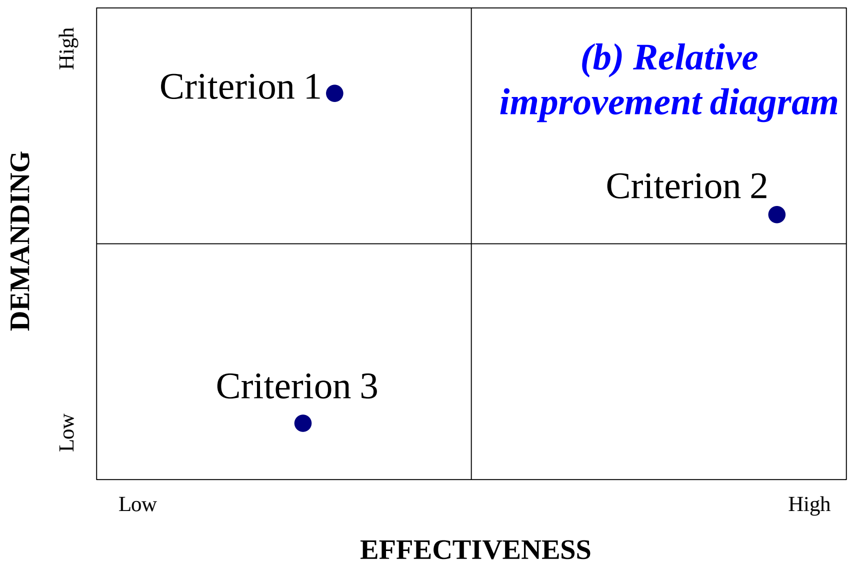


Numerical example (results)









Assumptions and inconsistencies

- Assumptions
  - The existence of an additive value function under certainty is based on the concept of preferential independence
- Inconsistencies
  - The most common problem is the lack of consistency for the collected data

Overall satisfaction	Criterion 1	Criterion 2	Criterion 3
Satisfied	Dissatisfied	Dissatisfied	Dissatisfied
Dissatisfied	Satisfied	Satisfied	Satisfied

Assumptions and inconsistencies

- Reasons for inconsistencies
  - There is not a consistent family of criteria
  - No ‘rational’ decision-makers
- In the preliminary stage of the MUSA method, a consistency control should be applied:
  - If inconsistencies occur in a small number of customers, these data should not be considered
  - In the opposite case, the set of assessed criteria should be reconsidered
- Other potential problems
  - Existence of distinguished customer groups with different preference value systems
  - Consider different customer segments

Evaluation of results (AFI)

- The fitting level of the MUSA method refers to the assessment of a collective value system for the set of customers (value functions, weights, etc.) with the minimum possible errors
- The Average Fitting Index (*AFI*) depends on the optimum error level and the number of customers as well:

$$AFI = 1 - \frac{F^*}{100M}$$

  where F^* is the minimum sum of errors of the basic LP
- AFI is normalized in [0, 1], and it is equal to 1 if $F^* = 0$ (case with zero errors).

Evaluation of results (alternative AFIs)

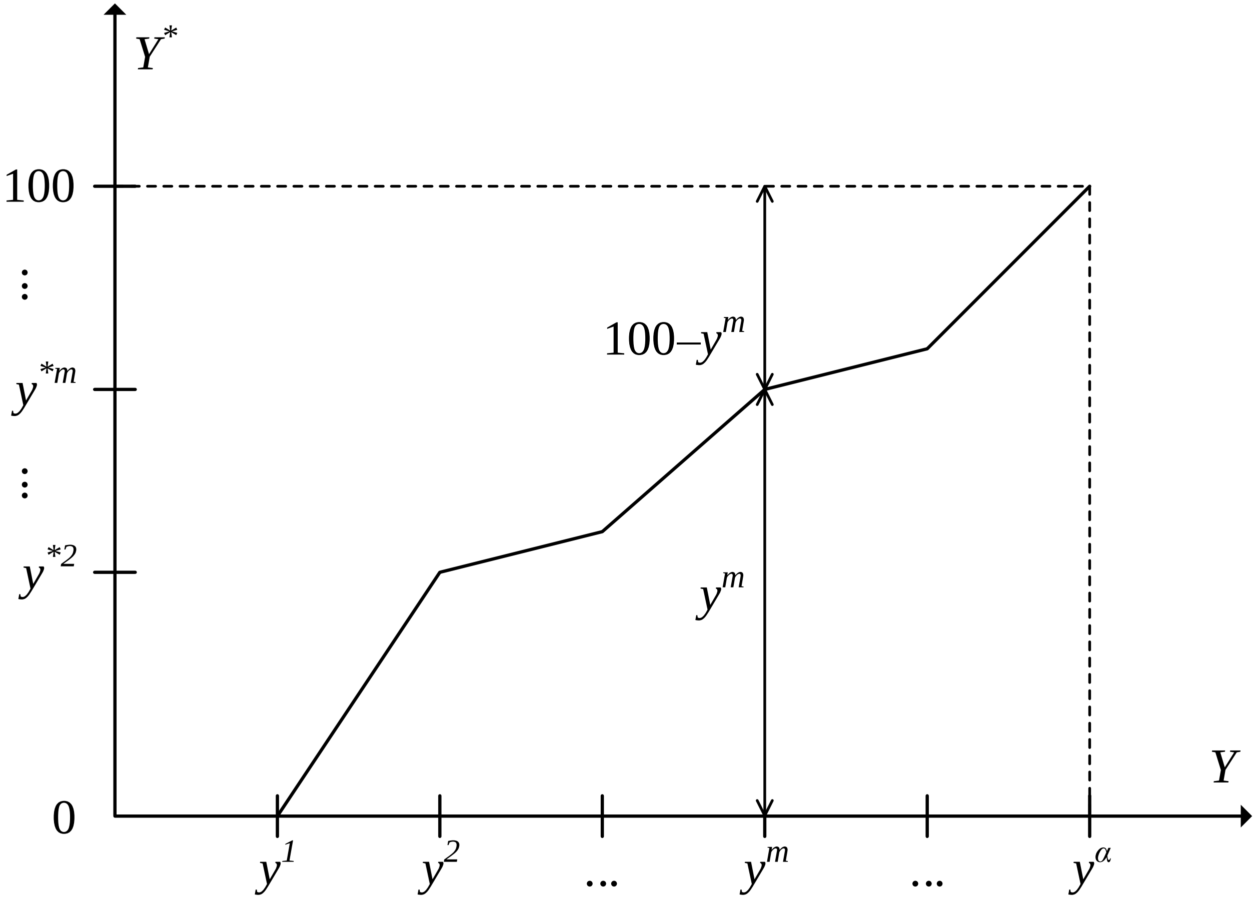
- Alternative fitting indicator based on the percentage of customers with zero error variables:

$$AFI' = \frac{M_0}{M}$$

- Alternative fitting indicator that takes into account the maximum values of the error variables for every global satisfaction level, as well as the number of customers that belongs to this level:

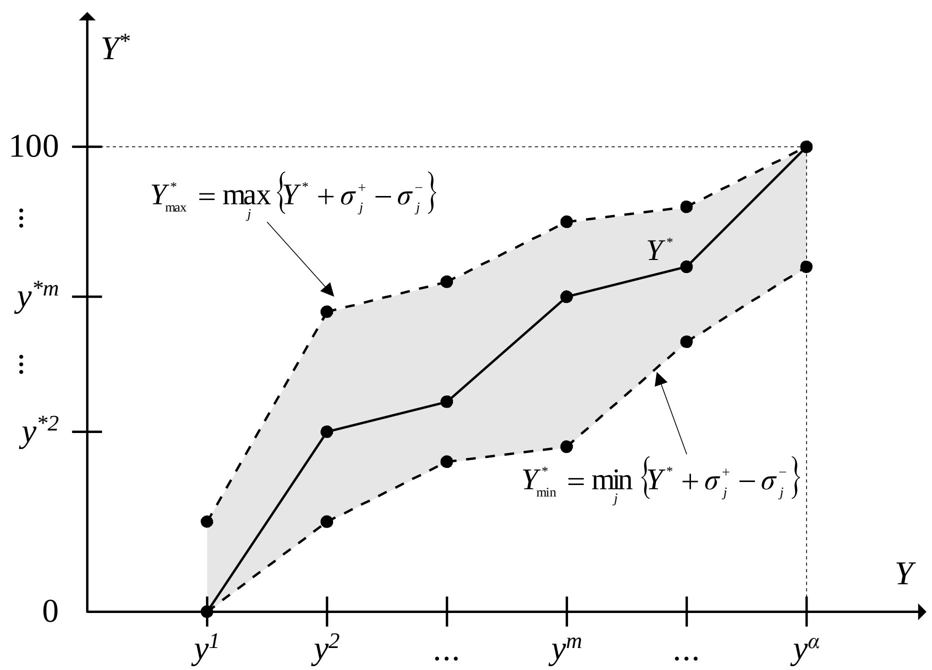
$$AFI'' = 1 - \frac{F^*}{M \sum_{m=1}^{\alpha} p^m \max\{y^{*m}, 100 - y^{*m}\}}$$

Evaluation of results (alternative AFIs)



Evaluation of results (other fitting indicators)

- **Variance diagram of the additive value curve**: using the estimated errors, the maximum and minimum satisfaction is calculated for each level of the ordinal satisfaction scale



Evaluation of results (other fitting indicators)

- **Prediction table of global satisfaction**: it refers to a classification for the observed and the predicted global satisfaction judgements

Actual Global Satisfaction Level	*Predicted Global Satisfaction Level*					
	$\tilde{y}^1$	$\tilde{y}^2$		$\tilde{y}^j$		$\tilde{y}^\alpha$
y^1	N_{11} R_{11} C_{11}	N_{12} R_{12} C_{12}	…	N_{1j} R_{1j} C_{1j}	…	$N_{1\alpha}$ $R_{1\alpha}$ $C_{1\alpha}$
y^2	N_{21} R_{21} C_{21}	N_{22} R_{22} C_{22}	…	N_{2j} R_{2j} C_{2j}	…	$N_{2\alpha}$ $R_{2\alpha}$ $C_{2\alpha}$
	⋮					⋮
y^i	N_{i1} R_{i1} C_{i1}	N_{i2} R_{i2} C_{i2}	…	N_{ij} R_{ij} C_{ij}	…	$N_{i\alpha}$ $R_{i\alpha}$ $C_{i\alpha}$
	⋮					⋮
y^α	N_{a1} R_{a1} C_{a1}	N_{a2} R_{a2} C_{a2}	…	N_{aj} R_{aj} C_{aj}	…	$N_{a\alpha}$ $R_{a\alpha}$ $C_{a\alpha}$

Evaluation of results (ASI)

- The stability of the MUSA method depends on the post-optimality analysis results
- During the post-optimality stage, *n* LPs are formulated and solved, which maximize repeatedly the weight of each criterion
- The Average Stability Index (*ASI*) is the mean value of the normalized standard deviation of the estimated weights:

$$ASI = 1 - \frac{1}{n}\sum_{i=1}^{n} \frac{\sqrt{n\sum_{j=1}^{n}\left(b_i^j\right)^2 - \left(\sum_{j=1}^{n} b_i^j\right)^2}}{100\sqrt{n-1}}$$

Numerical example (fitting)

- Fitting indicators
  - Since $F^* = 0$, we have $\sigma_j^+ = \sigma_j^- = 0, \forall j$
  - Thus, $AFI = AFI' = AFI'' = 1$
  - Prediction table

		Global satisfaction-Predicted		
		Dissatisfied	Neutral	Satisfied
Global satisfaction -Observed	Dissatisfied	30%	0%	0%
	Neutral	0%	40%	0%
	Satisfied	0%	0%	30%

Numerical example (stability)

- ASI=0.9177
- Post optimality results

	b_1	b_2	b_3
Max b_1	32.50	45.00	22.50
Max b_2	23.75	52.5	23.75
Max b_3	20.00	50.00	30.00

Extensions (strictly increasing value functions

- The basic MUSA method assumes that value functions are monotone non-decreasing
- Introducing strict preference conditions:

$$\begin{cases} y^{*m} < y^{*m+1} \\ x_i^{*k} < x_i^{*k+1} \end{cases} \Leftrightarrow \begin{cases} y^{m} \prec y^{m+1}, m = 1,2, \dots, \alpha - 1 \\ x_i^{k} \prec x_i^{k+1}, k = 1,2, \dots, \alpha_i - 1 \; i = 1,2, \dots, n \end{cases}$$

- Additional constraints

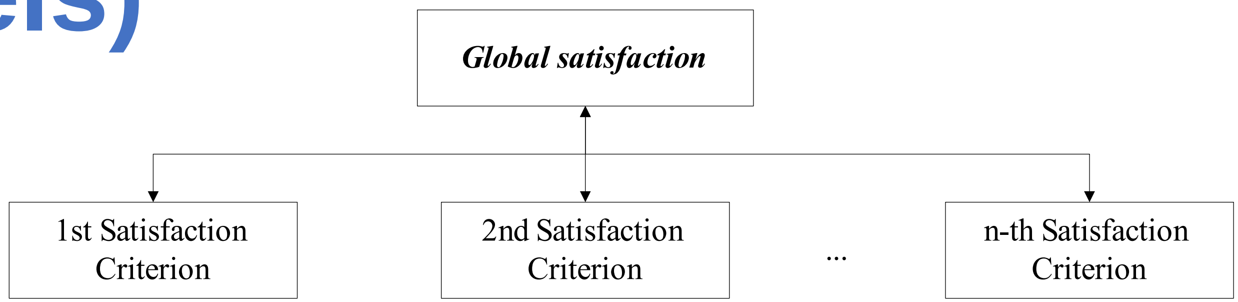
$$\begin{cases} y^{*m+1} - y^{*m} \geq \gamma, m = 1,2, \dots, \alpha - 1 \\ x_i^{*k+1} - x_i^{*k} \geq \gamma_i, k = 1,2, \dots, \alpha_i - 1 \; i = 1,2, \dots, n \end{cases}$$

where γ, γ_i are preference thresholds for the value functions Y^* and X_i^*

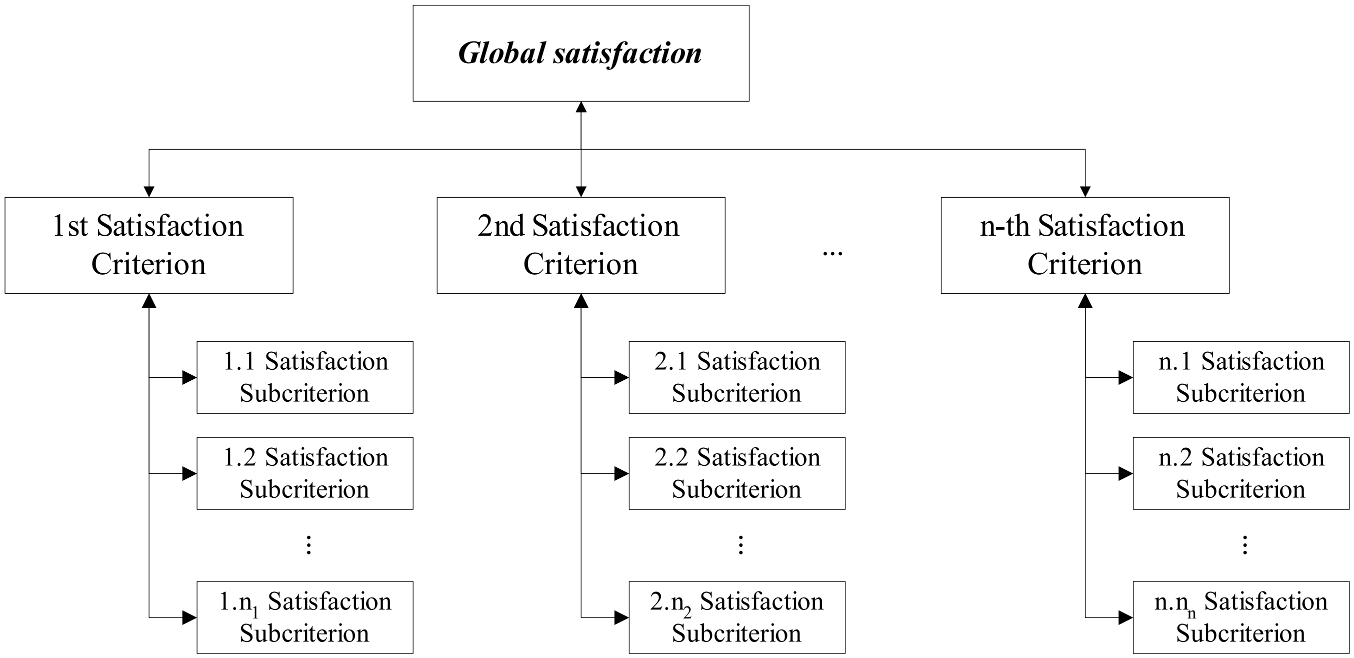
Extensions (multiple criteria levels)

- In several cases multiple criteria levels should be assessed
  - The 1st criteria level contains the general satisfaction dimensions (e.g. personnel)
  - The 2nd criteria level refers to the analytical dimensions of the main criteria (e.g. skills and knowledge of personnel).
- The assessed hierarchical structure should satisfy the properties of a consistent family of criteria
- In this extension, the MUSA method can be similarly reformulated in a LP problem following the presented principles and basic concepts

Extensions (multiple criteria levels)



(α) 1 level of satisfaction criteria



(β) 2 levels of satisfaction criteria

Extensions (multiple criteria levels)

Additional variables of the model (2 criteria levels)	
n_i	: number of subcriteria for the i-th criterion
X_{ij}	: client's satisfaction according to the j-th subcriterion of the i-th criterion ($j=1, 2, \ldots, n_i$, $i=1, 2, \ldots, n$)
α_{ij}	: number of satisfaction levels for the j-th subcriterion of the i-th criterion
x_{ij}^k	: the k-th satisfaction level for the j-th subcriterion of the i-th criterion ($k=1, 2, ..., \alpha_{ij}$)
X_{ij}^*	: value function of X_{ij}
x_{ij}^{*k}	: value of the x_{ij}^k satisfaction level
b_i	: weight for the i-th criterion
b_{ij}	: weight for the j-th subcriterion of the i-th criterion

Extensions (multiple criteria levels)

- In this case the ordinal regression analysis equations for the MUSA method have as follows:

$$Y^* = \sum_{i=1}^{n} b_i X_i^* \text{ with } \sum_{i=1}^{n} b_i = 1$$

$$X_i^* = \sum_{j=1}^{n_i} b_{ij} X_{ij}^* \text{ with } \sum_{j=1}^{n_i} b_{ij} = 1 \text{ for } i = 1,2,\dots,n$$

  where Y^*, X_i^*, X_{ij}^* are normalized in [0, 100]

- Similarly to the 1 criteria level problem, the additional variables of the LP problem refer to the repeated steps of the subcriteria value functions

$$w_{ijk} = b_i b_{ij} x_{ij}^{*k+1} - b_i b_{ij} x_{ij}^{*k}, i = 1,2,\dots,n$$
$$j = 1,2,\dots,n_i \; k = 1,2,\dots,\alpha_{ij} - 1$$

Extensions (multiple criteria levels)

$$
\begin{cases}
[\min] F = \sum_{q=1}^{M} \sigma_q^+ + \sigma_q^- + \sum_{q=1}^{M}\sum_{i=1}^{n} \sigma_{qi}^+ + \sigma_{qi}^- \\
\text{subject to} \\
\sum_{i=1}^{n}\sum_{k=1}^{t_{qi}-1} w_{ik} - \sum_{m=1}^{t_q-1} z_m - \sigma_q^+ + \sigma_q^- = 0 \quad \text{for } q=1,2,\ldots,M \\
\sum_{j=1}^{n_i}\sum_{k=1}^{t_{qij}-1} w_{ijk} - \sum_{k=1}^{t_{qi}-1} w_{ik} - \sigma_{qi}^+ + \sigma_{qi}^- = 0 \quad \text{for } i=1,2\ldots,n \text{ and } q=1,2,\ldots,M \\
\sum_{m=1}^{\alpha-1} z_m = 100 \\
\sum_{i=1}^{n}\sum_{k=1}^{\alpha_i-1} w_{ik} = 100 \\
\sum_{i=1}^{n}\sum_{j=1}^{n_i}\sum_{k=1}^{\alpha_{ij}-1} w_{ijk} = 100 \\
z_m \geq 0,\ w_{ik} \geq 0,\ w_{ijk} \geq 0 \qquad \forall\, m,i,j,k \\
\sigma_q^+ \geq 0,\ \sigma_q^- \geq 0,\ \sigma_{qi}^+ \geq 0,\ \sigma_{qi}^- \geq 0 \qquad \forall\, q,i
\end{cases}
$$

where t_q, t_{qi}, t_{qij} is the global, criteria and subcriteria judgement of the q-th customer respectively.

Extensions (alternative post-optimality analyses)

Extension	Objective function	Remarks
Gen. MUSA	$[\max]F' = b_i$	Basic model with preference thresholds
MUSA I	$[\max]F' = b_i$, $[\min]F' = b_i$	Maximization-minimization of weights
MUSA II	$[\max]F' = \gamma$, $[\max]F' = \gamma_i$	Maximization of preference thresholds
MUSA III	$[\max]F' = z_m$, $[\max]F' = w_{ik}$	Maximization of value functions' successive steps
MUSA IV	$[\max]F' = m_e$	Minimization of the L_∞ norm of errors

Numerical example (alternative post-optimality analyses)

	Basic MUSA method ($\gamma = \gamma_i = 0$)	Generalized MUSA method ($\gamma = \gamma_i = 2$)	MUSA I	MUSA II	MUSA III	MUSA IV
b_1	25.42	25.58	26.46	26.21	26.51	25.00
b_2	49.17	48.83	48.75	49.25	49.06	50.00
b_3	25.42	25.58	24.79	24.54	24.43	25.00

Extensions (robustness)

- Generic robust approach:
  - Infer a collective preference model
  - Calculate a robustness measure (e.g., ASI)
  - Improve the robustness of the model (i.e., consider additional information):
    - Preferences on criteria importance (Grigoroudis and Siskos, 2010)
    - Interaction among criteria (Angilella et al., 2014)
    - Additional properties regarding the provided results (i.e., average satisfaction/demanding indices)

Extensions (additional properties)

- Average satisfaction indices:

$$S = \sum_{i=1}^{n} b_i S_i \Rightarrow \sum_{m=1}^{\alpha} p^m y^{*m} = \sum_{i=1}^{n} b_i \sum_{k=1}^{\alpha_i} p_i^k x_i^{*k} \Rightarrow \sum_{m=2}^{\alpha} p^m \sum_{t=1}^{m-1} z_t = \sum_{i=1}^{n} \sum_{k=2}^{\alpha_i} p_i^k \sum_{t=1}^{k-1} w_{it}$$

- Average demanding indices:

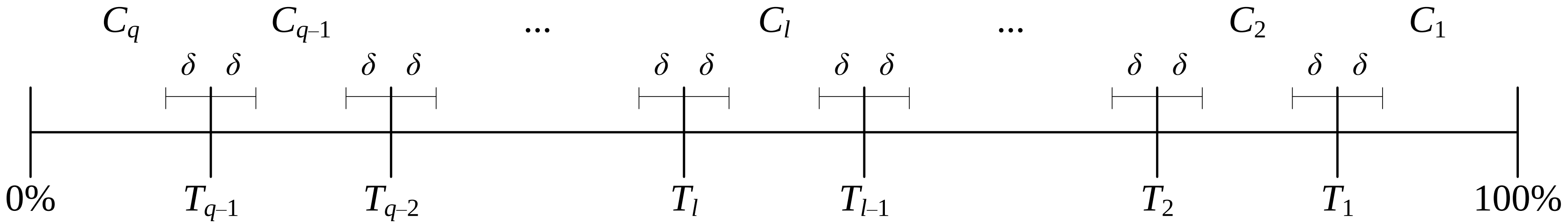
$$D = \sum_{i=1}^{n} b_i D_i \Rightarrow \frac{\sum_{m=1}^{\alpha-1} 100(m-1) - (\alpha-1) \sum_{t=1}^{m-1} z_t}{\alpha(\alpha-1)} = \sum_{i=1}^{n} \frac{\sum_{k=1}^{\alpha_i-1} (k-1) \sum_{t=1}^{\alpha_i-1} w_{it} - (\alpha_i - 1) \sum_{t=1}^{k-1} w_{it}}{\alpha_i(\alpha_i - 1)}$$

Extensions (criteria importance)

- A customer satisfaction survey may include, besides the usual performance questions, preferences about the importance of the criteria
- Using such questions, customers are asked either to judge the importance of a satisfaction criterion using a predefined ordinal scale, or rank the set of satisfaction criteria according to their importance
- Based on such importance questions, each one of the satisfaction criteria can be placed in one of the following categories C_1, C_2, …, C_q, where C_1 is the most important criterion class and C_q is the less important criterion class. Considering that C_l, with l the class index, are ordered in a 0-100% scale, there are T_q–1 thresholds, which define the rank and, therefore, label each one of the classes

Extensions (criteria importance)

- The evaluation of preference importance classes C_l is similar to the estimation of thresholds T_l



Extensions (WORT model)

$$[\min] \quad F_2 = \sum_j \sum_i S_{ij}^{+} + S_{ij}^{-}$$

$$\left.\begin{array}{l}
\displaystyle\sum_{t=1}^{a_i-1} w_{it} - 100\ T_1 - \delta + S_{ij}^{-} > 0, \quad \hat{b}_{ij} \in C_1 \\
\left.\begin{array}{l}
\displaystyle\sum_{t=1}^{a_i-1} w_{it} - 100\ T_{l-1} + \delta - S_{ij}^{+} < 0 \\
\displaystyle\sum_{t=1}^{a_i-1} w_{it} - 100\ T_l - \delta + S_{ij}^{-} \geq 0
\end{array}\right\}, \hat{b}_{ij} \in C_l,\ l = 2,\ldots, q-1 \\
\displaystyle\sum_{t=1}^{a_i-1} w_{it} - 100\ T_{q-1} + \delta - S_{ij}^{+} < 0, \quad \hat{b}_{ij} \in C_q
\end{array}\right\} \quad \forall \quad i = 1,2,\ldots, n \quad \text{και} \quad j = 1,2,\ldots, M$$

$$\sum_{i=1}^{n} \sum_{k=1}^{a_i-1} w_{ik} = 100$$

$$T_{q-1} \quad \geq \quad \lambda$$

$$T_{q-2} - T_{q-1} \quad \geq \quad \lambda$$

$$\vdots$$

$$T_1 - T_2 \quad \geq \quad \lambda$$

$$w_{ik},\ S_{ij}^{+},\ S_{ij}^{-} \geq 0,\ \forall\ i,\ j,\ k$$

Extensions (criteria importance)

- Using together customers' performance and importance judgments, an extension of the MUSA method may be modeled as a Multiobjective Linear Programming (MOLP) problem

$$\begin{cases} [\min] F_1 = \sum_{j=1}^{M} \sigma_j^+ + \sigma_j^- \\ [\min] F_2 = \sum_{i=1}^{n} \sum_{j=1}^{M} S_{ij}^+ + S_{ij}^- \\ \text{subject to} \\ \text{all the constraints of the basic MUSA and the WORT models} \end{cases}$$

Numerical example (criteria importance)

Importance Judgments		
Criterion 1	**Criterion 2**	**Criterion 3**
Important	Important	Important
Important	Very important	Unimportant
Important	Important	Important
Important	Very important	Unimportant
Important	Very important	Important
Important	Very important	Unimportant
Very important	Important	Unimportant
Very important	Important	Unimportant
Important	Very important	Important
Important	Very important	Unimportant
Important	Very important	Important
Important	Important	Important
Important	Very important	Unimportant
Very important	Important	Important
Important	Very important	Important
Important	Very important	Important
Important	Very important	Important
Unimportant	Important	Very important
Important	Important	Important
Important	Very important	Unimportant

Numerical example (criteria importance)

	Basic MUSA model	Compromise programming	Global criterion	Heuristic method
Criterion 1 weight	25.42%	36.63%	36.04%	36.30%
Criterion 2 weight	49.17%	36.69%	37.27%	37.02%
Criterion 3 weight	25.42%	26.68%	26.69%	26.68%
ASI	**91.77%**	**99.98%**	**98.81%**	**99.31%**

Extensions (additional properties)

- Add additional constraints to the basic LP formulation.
- If necessary consider these constraints in the following order:
  - Constraint for average satisfaction indices
  - Constraint for average demanding indices
- In the general case, these constraints may lead to infeasible solutions, thus:
  - They should be modeled using a double error variable
  - In this case, a MOLP approach may be applied (e.g., compromise programming)

Numerical example (additional properties)

	MUSA method + Constraint for average satisfaction indices	MUSA method + Constraints for average satisfaction/ demanding indices
ASI	+11.06%	+11.98%

Concluding remarks

- The MUSA method is based on the principles of aggregation-disaggregation approach and linear programming modelling.
- Main advantages of the method:
  - It fully considers the qualitative form of customers' judgements and preferences, as expressed in a customer satisfaction survey
  - The post-optimality analysis stage gives the ability to achieve a sufficient stability level
  - The provided results are focused not only on the descriptive analysis of customer satisfaction data, but they are also able to assess an integrated benchmarking system
  - It is based on a very flexible modeling

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