Decision Modelling II

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Outline



- Preferences Handling
 - Problem Setting
 - Basics
 - Preference Modeling
 - Preference Aggregation
- 2 Social Choice Theory
- Borda and Condorcet
 - The Borda path
 - The Condorcet path

Conclusions

Problem Setting Basics Preference Modeling Preference Aggregation



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- Preferences are "rational" desires.
- Preferences are at the basis of any decision aiding activity.
- There are no decisions without preferences.
- Preferences, Values, Objectives, Desires, Utilities, Beliefs,

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Preference Statements:

- I like red shoes.
- I do not like brown sugar.
- I prefer Maria to Mario.
- I do not want tea with milk.
- Cost is more important than safety.
- I prefer flying to Athens than having a suite at Istanbul.
- I prefer red wine only if there is no fish plate available.

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Preference Statements:

Four issues:

Relative vs Absolute statements Single vs Multi-attribute statements Positive vs Negative statements First vs Second order statements

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What are the problems?

- How to learn preferences?
- How to model preferences?
- How to aggregate preferences?
- How to use preferences for recommending?

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Binary relations

- \succeq : binary relation on a set (A).
- $\succeq \subseteq A \times A \text{ or } A \times P \cup P \times A.$
- \succeq is reflexive.

What is that?

If $x \succeq y$ stands for x is at least as good as y, then the asymmetric part of $\succeq (\succ: x \succeq y \land \neg(y \succeq x))$ stands for strict preference. The symmetric part stands for indifference $(\sim_1: x \succeq y \land y \succeq x)$ or incomparability $(\sim_2: \neg(x \succeq y) \land \neg(y \succeq x)).$

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More binary relations

 We can further separate the asymmetric (symmetric) part in more relations representing hesitation or intensity of preference.

$$\succ = \succ_1 \cup \succ_2 \cdots \succ_n$$

- We can get rid of the symmetric part since any symmetric relation can be viewed as the union of two asymmetric relations and the identity.
- We can also have valued relations such that:
 v(x ≻ y) ∈ [0, 1]

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Binary relations properties

Binary relations have specific properties such as:

- Irreflexive: $\forall x \neg (x \succ x);$
- Asymmetric: $\forall x, y \ x \succ y \rightarrow \neg (y \succ x);$
- Transitive: $\forall x, y, z \ x \succ y \land y \succ z \rightarrow x \succ z$;
- Ferrers; $\forall x, y, z, w \ x \succ y \land z \succ w \rightarrow x \succ w \lor z \succ y$;

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Numbers

$x \succeq y \quad \Leftrightarrow \quad \Phi(x,y) \ge 0$

where:

 $\Phi : A \times A \mapsto \mathbb{R}$. Simple case $\Phi(x, y) = f(x) - f(y); f : A \mapsto \mathbb{R}$

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Preference Structures

A preference structure

is a collection of binary relations $\sim_1, \dots \sim_m, \succ_1, \dots \succ_n$ such that:

- they are pair-disjoint;
- $\sim_1 \cup \cdots \sim_m \cup \succ_1 \cup \cdots \succ_n = A \times A;$
- \sim_i are symmetric and \succ_j are asymmetric;
- possibly they are identified by their properties.

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\sim_1, \sim_2, \succ Preference Structures

Independently from the nature of the set *A* (enumerated, combinatorial etc.), consider $x, y \in A$ as whole elements. Then:

If \succeq is a weak order then:

 \succ is a strict partial order, \sim_1 is an equivalence relation and \sim_2 is empty.

If \succeq is an interval order then:

 \succ is a partial order of dimension two, \sim_1 is not transitive and \sim_2 is empty.

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Problem Setting Basics Preference Modeling Preference Aggregation

$\sim_1, \sim_2, \succ_1 \succ_2$ Preference Structures

If \succeq is a *PQI* interval order then:

 \succ_1 is transitive, \succ_2 is quasi transitive, \sim_1 is asymmetrically transitive and \sim_2 is empty.

If \succeq is a pseudo order then:

 \succ_1 is transitive, \succ_2 is quasi transitive, \sim_1 is non transitive and \sim_2 is empty.

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Problem Setting Basics Preference Modeling Preference Aggregation

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Problem Setting Basics Preference Modeling Preference Aggregation

What characterises such structures?

Characteristic Properties

Weak Orders are complete and transitive relations. Interval Orders are complete and Ferrers relations.

Numerical Representations

w.o. $\Leftrightarrow \exists f : A \mapsto \mathbb{R} : x \succeq y \leftrightarrow f(x) \ge f(y)$ i.o. $\Leftrightarrow \exists f, g : A \mapsto \mathbb{R} : f(x) > g(x); x \succeq y \leftrightarrow f(x) \ge g(y)$

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Problem Setting Basics Preference Modeling Preference Aggregation

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Problem Setting Basics Preference Modeling Preference Aggregation

More about structures

Characteristic Properties

PQI Interval Orders are complete and generalised Ferrers relations.

Pseudo Orders are coherent bi-orders.

Numerical Representations

 $\begin{aligned} & PQl \text{ i.o. } \Leftrightarrow \exists f,g: A \mapsto \mathbb{R} : f(x) > g(x); \\ & x \succ_1 y \iff g(x) > f(y); x \succ_2 y \iff f(x) > f(y) > g(x) \\ & P.O. \ \Leftrightarrow \exists f,t,g: A \mapsto \mathbb{R} : f(x) > t(x) > g(x); \\ & x \succ_1 y \iff g(x) > f(y); x \succ_2 y \iff g(x) > t(y) \end{aligned}$

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More about structures

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PQI Interval Orders are complete and generalised Ferrers relations.

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Numerical Representations

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Problem Setting Basics Preference Modeling Preference Aggregation

What if A is multi-attribute described?

$$x = \langle x_1 \cdots x_n \rangle \quad y = \langle y_1 \cdots y_n \rangle$$

$x \succeq y \quad \Leftrightarrow \quad \Phi([u_1(x_1) \cdots u_n(n)], [u_1(y_1) \cdots u_n(y_n)] \ge 0$

A special case is when Φ is increasing to its first *n* arguments and decreasing to the following *n* arguments: it then can be an additive function. See more in conjoint measurement theory.

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Problem Setting Basics Preference Modeling Preference Aggregation

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Preferences Handling Problem Social Choice Theory Basics Borda and Condorcet Prefere Conclusions Prefere

Problem Setting Basics Preference Modeling Preference Aggregation

The Problem

Suppose we have *n* ordering relations $\succeq_1 \cdots \succeq_n$ on the set *A*. We are looking for an overall ordering relation \succeq on *A* "representing" the different orders.



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Borda vs. Condorcet

Four candidates and seven examiners with the following preferences.

	а	b	С	d	е	f	g
Α	1	2	4	1	2	4	1
В	2	3	1	2	3	1	2
С	3	1	3	3	1	2	3
D	4	4	2	4	4	3	4

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Borda vs. Condorcet

Four candidates and seven examiners with the following preferences.

	а	b	С	d	е	f	g	B(x)
Α	1	2	4	1	2	4	1	15
В	2	3	1	2	3	1	2	14
С	3	1	3	3	1	2	3	16
D	4	4	2	4	4	3	4	25

The Borda count gives B>A>C>D

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Α	1	2	3	1	2	3	1	13
В	2	3	1	2	3	1	2	14
С	3	1	2	3	1	2	3	15

If D is not there then A>B>C, instead of B>A>C

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Borda vs. Condorcet

Four candidates and seven examiners with the following preferences.

	а	b	С	d	е	f	g
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С	3	1	2	3	1	2	3

The Condorcet principle gives A>B>C>A !!!!

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Arrow's Theorem

Given *N* rational voters over a set of more than 3 candidates can we found a social choice procedure resulting in a social complete order of the candidates such that it respects the following axioms?

- Universality: the method should be able to deal with any configuration of ordered lists;
- Unanimity: the method should respect a unanimous preference of the voters;
- Independence: the comparison of two candidates should be based only on their respective standings in the ordered lists of the voters.

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There is only one solution: the dictator!!

If we add no-dictatorship among the axioms then there is no solution.

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Gibbard-Satterthwaite's Theorem

When the number of candidates is larger than two, there exists no aggregation method satisfying simultaneously the properties of universal domain, non-manipulability and non-dictatorship.

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Why MCDA is not Social Choice?

Social Choice	MCDA
Total Orders	Any type of order
Equal importance	Variable importance
of voters	of criteria
As many voters	Few coherent
as necessary	criteria
No prior	Existing prior
information	information

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Counting values

$$x \succeq y \Leftrightarrow \sum_j r_j(x) \ge \sum_j r_j(y)$$

What do we need to know?

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Counting values

$$x \succeq y \Leftrightarrow \sum_j r_j(x) \ge \sum_j r_j(y)$$

What do we need to know?

the primitives: $\succeq_j \subseteq A \times A$ Differences of preferences:

- $(xy)_1 \succcurlyeq (zw)_1$
- $(xy)_1 \succcurlyeq (zw)_2$

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How do we learn that?

- Directly through a standard protocol.
- Indirectly:
 - through pairwise comparisons (AHP, MACBETH etc.);
 - through learning from examples (regression, rough sets, decision trees etc.).

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Is this sufficient?

NO!

Are preferences independent? $r \succ w$ $f \succ m$ But *rf* is not better than *wf* ...

Non linear aggregation procedures

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What is the output?

• Value functions on each criterion.

- A global value function.
- Rankings, choices, but also ratings if relevant reference points are provided on the value function.

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Counting preferences

$$x \succeq y \Leftrightarrow H_{xy} \ge H_{yx}$$

What do we need to know?

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Counting preferences

$$x \succeq y \Leftrightarrow H_{xy} \ge H_{yx}$$

What do we need to know?

the primitives: $\succeq_j \subseteq A \times A$ An ordering relation on 2^{\succeq_j}

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How do we learn that?

- Preferences are "given".
- Preferences on 2[≥]_j:
 - directly;
 - coalition games;
 - learning from examples.

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Is this sufficient?

NO!

- The relation \succeq is not an ordering relation.
- In order to do so we transform the graph induced by \succeq .

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Is this sufficient?

NO!

- The relation \succeq is not an ordering relation.
- We need to construct an ordering relation ≽ "as near as possible" to *b*.
- In order to do so we transform the graph induced by \succeq .

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General idea: coalitions

Given a set A and a set of \succeq_i binary relations on A (the criteria) we define:

$$x \succeq y \Leftrightarrow C^+(x,y) \trianglerighteq C^+(y,x)$$
 and $C^-(x,y) \trianglelefteq C^-(y,x)$

where:

- $C^+(x, y)$: "importance" of the coalition of criteria supporting *x* wrt to *y*.

- $C^{-}(x, y)$: "importance" of the coalition of criteria against *x* wrt to *y*.

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How it works? 1

Additive Positive Importance

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How it works? 1

Additive Positive Importance

$$\mathcal{C}^+(x,y) = \sum_{j \in J^{\pm}} w_j^+$$

where: w_j^+ : "positive importance" of criterion *i* $J^{\pm} = \{h_j : x \succeq_j y\}$

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How it works? 1

Additive Positive Importance

$$\mathcal{C}^+(x,y) = \sum_{j \in J^{\pm}} w_j^+$$

where: w_j^+ : "positive importance" of criterion *i* $J^{\pm} = \{h_j : x \succeq_j y\}$

Then we can fix a majority threshold $\boldsymbol{\delta}$ and have

$$x \succeq^+ y \Leftrightarrow C^+(x,y) \ge \delta$$

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How it works? 1

Additive Positive Importance

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Where "positive importance" comes from?

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How it works? 2

Max Negative Importance



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How it works? 2

Max Negative Importance

$$C^-(x,y) = \max_{j\in J^-} w_j^-$$

where:

 w_j^- : "negative importance" of criterion *i* $J^- = \{h_j : v_j(x, y)\}$

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How it works? 2

Max Negative Importance

$$C^-(x,y) = \max_{j\in J^-} w_j^-$$

where:

 w_j^- : "negative importance" of criterion *i* $J^- = \{h_j : v_j(x, y)\}$

Then we can fix a veto threshold γ and have

$$\mathbf{x} \succeq^{-} \mathbf{y} \Leftrightarrow \mathbf{C}^{-}(\mathbf{x}, \mathbf{y}) \geq \gamma$$

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How it works? 2

Max Negative Importance

$$C^-(x,y) = \max_{j\in J^-} w_j^-$$

where:

 w_j^- : "negative importance" of criterion *i* $J^- = \{h_j : v_j(x, y)\}$

Then we can fix a veto threshold γ and have

$$x \succeq^{-} y \Leftrightarrow C^{-}(x, y) \geq \gamma$$

Where "negative importance" comes from?

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The United Nations Security Council

Positive Importance

15 members each having the same positive importance $w_j^+ = \frac{1}{15}, \, \delta = \frac{9}{15}.$

Negative Importance

10 members with 0 negative importance and 5 (the permanent members) with $w_i^- = 1$, $\gamma = 1$.

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15 members each having the same positive importance $w_j^+ = \frac{1}{15}, \, \delta = \frac{9}{15}.$

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10 members with 0 negative importance and 5 (the permanent members) with $w_i^- = 1$, $\gamma = 1$.

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Outranking Principle

$$x \succeq y \Leftrightarrow x \succeq^+ y \text{ and } \neg(x \succeq^- y)$$

Thus:

$$x \succeq y \Leftrightarrow C^+(x,y) \ge \delta \land C^-(x,y) < \gamma$$

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Outranking Principle

$$x \succeq y \Leftrightarrow x \succeq^+ y \text{ and } \neg(x \succeq^- y)$$

Thus:

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NB

The relation \succeq is not an ordering relation. Specific algorithms are used in order to move from \succeq to an ordering relation \succcurlyeq

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What is importance?

Where w_j^+ , w_j^- and δ come from?

Further preferential information is necessary, usually under form of multi-attribute comparisons. That will provide information about the decisive coalitions.

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What is importance?

Where w_j^+ , w_j^- and δ come from?

Further preferential information is necessary, usually under form of multi-attribute comparisons. That will provide information about the decisive coalitions.

Example

Given a set of criteria and a set of decisive coalitions (J^{\pm}) we can solve:

 $\max \delta \\ \text{subject to} \\ \sum_{j \in J^{\pm}} w_j \ge \delta \\ \sum_j w_j = 1$

Conclusions

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And the final ranking?

•
$$x \succcurlyeq y \Leftrightarrow o(x) - i(x) \ge o(y) - i(y)$$

● Recursively constructing ≽:

•
$$[x]_1 = \{x \in A : \neg \exists y \ y \succeq x\}$$

 $[x]_i = \{x \in A \setminus \bigcup_{i=1} [x] : \neg \exists y \ y \succeq x\}$

•
$$[X]_n = \{x \in A : \neg \exists y \ x \succeq y\}$$

 $[X]_i = \{x \in A \setminus \bigcup_{n=i} [x] : \neg \exists y \ x \succeq y\}$

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Rating

What if we have preference relations $\succeq_j \subseteq A \times P \cup P \times A$? The global preference relation remains the same.

- pessimistic rating
 - *x* is iteratively compared with $p_t \cdots p_1$,
 - as soon as $(x \succeq p_h)$ is established, assign x to category c_h .
- optimistic rating
 - *x* is iteratively compared with $p_1 \cdots p_t$,

- as soon as is established $(p_h \succeq x) \land \neg (x \succeq p_h)$ then assign *x* to category c_{h-1} .

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What is the output?

• A global preference relation including incomparabilities.

- An explicit representation of hesitation.
- Robust Rankings, Choices and Ratings.

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- Preferences are ultimately binary relations. Numerical representations are useful, but not strictly necessary.
- "Weights" do not exist independently. They are not primitives, but second order models.
- There is no universal preference aggregation procedure and will never exist one. We always need to justify why we adopt that precise one and for which purpose.
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