Middle East Technical University

Multiple Criteria Decision Making: Interactive Approaches

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Outline

- Problem Definition
- Classification
- Choice (Discrete alternative) problem
- Design (Continuous) problem
- MOCO
- Resources

Problem Definition

$$(P) \quad \text{"}Min\text{"} \; \{z_1(x), \ldots, z_p(x)\}$$

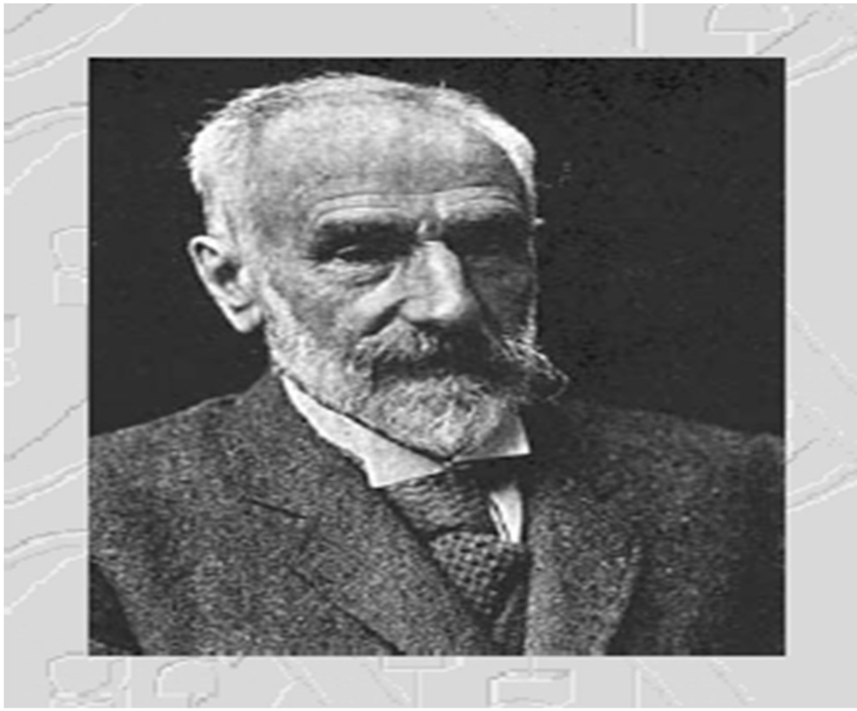
$$x \in X$$

where

x : *decision vector*

X : *solution space*

z_i : *ith objective function*



Francis Ysidro Edgeworth
1881

Vilfredo Pareto
1896

Notion of Edgeworth-Pareto Optimal

Efficient solution

- $\mathbf{x}$ is an efficient solution if there exists no $\mathbf{x}'$ such that

$$z_i(\mathbf{x}') \leq z_i(\mathbf{x}) \; i=1,\ldots,p \text{ and}$$
$$z_i(\mathbf{x}') < z_i(\mathbf{x}) \text{ for at least one } i.$$

- If x is efficient then $(z_1(x),\ldots,z_p(x))$ is said to be nondominated.
- Remark: "Best" solution is an efficient solution.

Efficient set/frontier

- The set of all efficient solutions form the efficient set/frontier
- The set of all nondominated points form the nondominated set/frontier

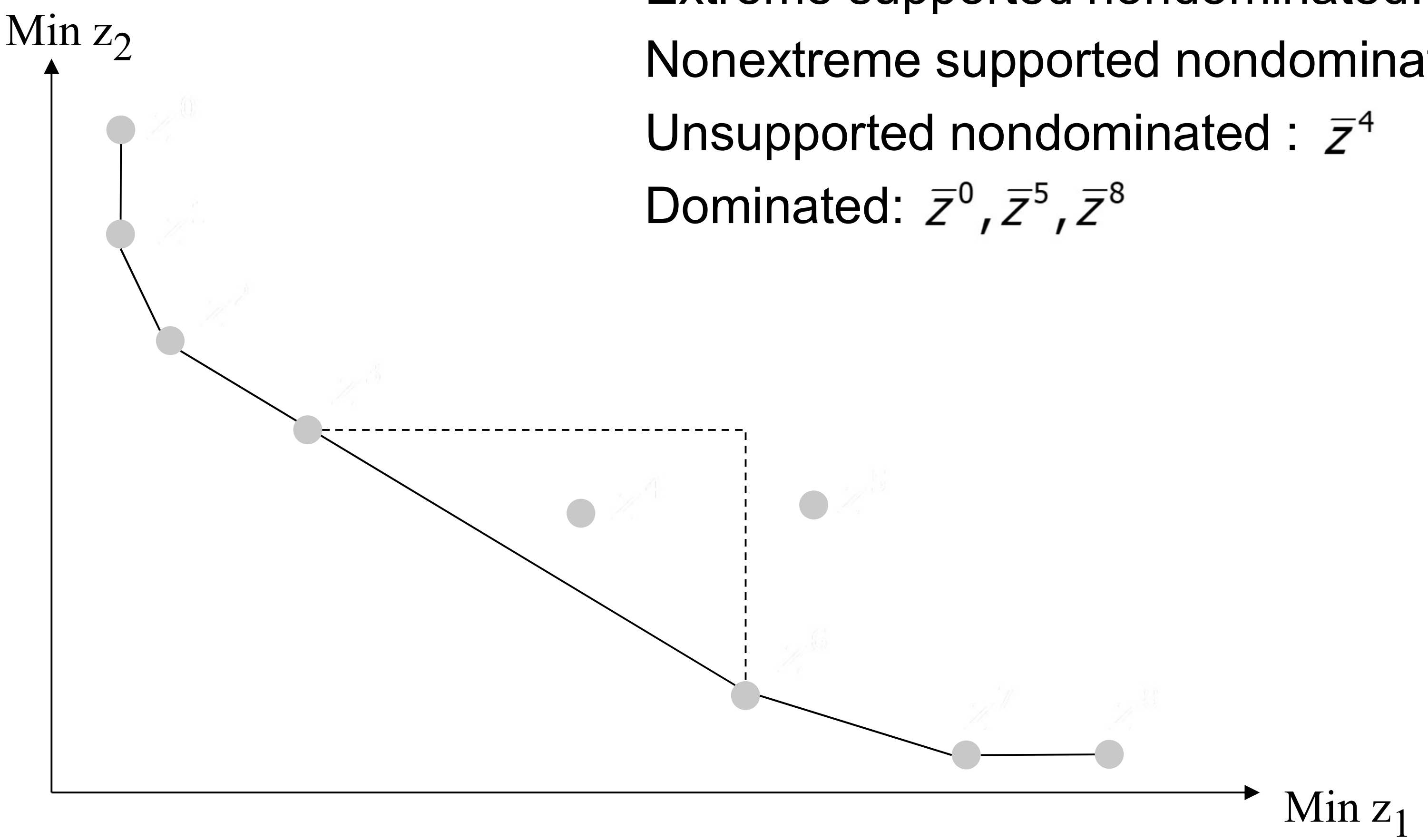
Solution Types

Extreme supported nondominated: $\bar{z}^1, \bar{z}^2, \bar{z}^6, \bar{z}^7$

Nonextreme supported nondominated: $\bar{z}^3$

Unsupported nondominated : $\bar{z}^4$

Dominated: $\bar{z}^0, \bar{z}^5, \bar{z}^8$



Classification

- If $X=\{X_1,\ldots,X_n\}$ then
  Choice (*Discrete Alternative) Problem*
  where $X_i=(x_{i1},\ldots,x_{ip})$

- If $X=\{\mathbf{x}: g_j(\mathbf{x}) \le b_j \; j=1,\ldots,m\}$ then Design (*Continuous Solution Space) Problem*

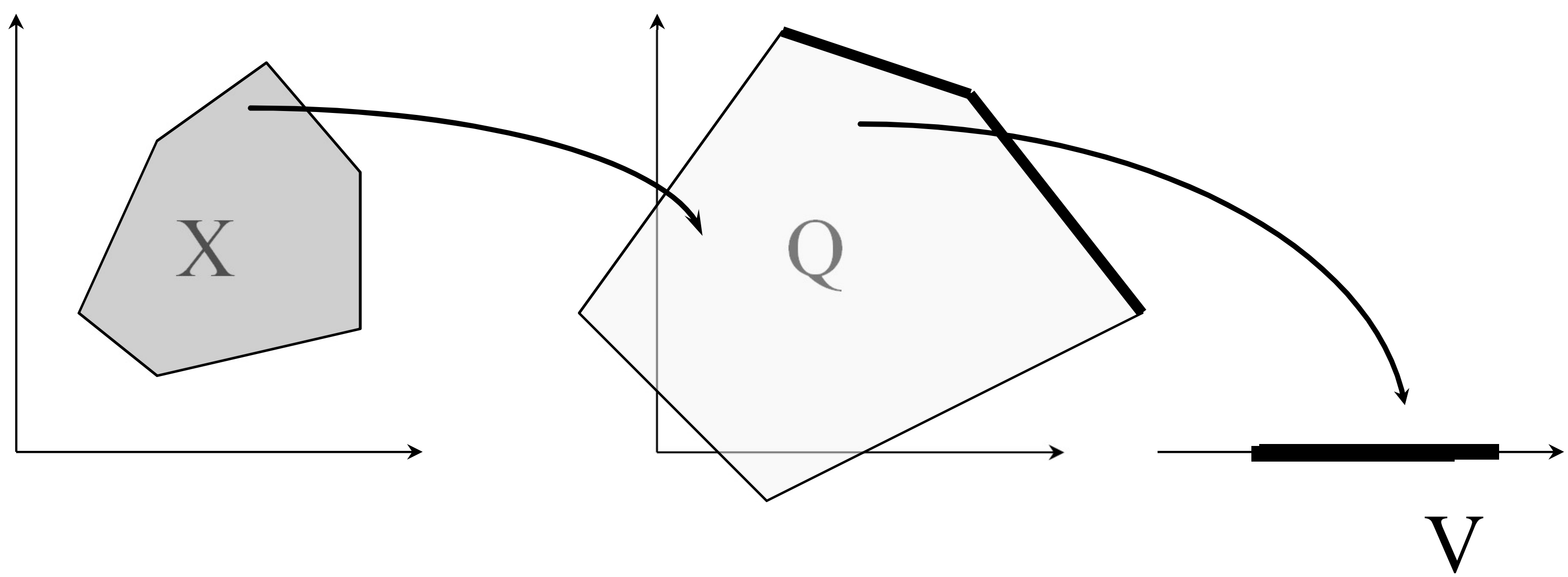
- If X is discrete and "large" then
  Combinatorial Problem

MOCO

- X is discrete and “large”
- Grows fast with problem size
- Hope of solving to optimality reduces as fast with problem size

Basic Concepts: Variable, Criterion and Value Space

Continuous Solution Space



Basic Concepts: Variable, Criterion and Value Space

Discrete Alternative

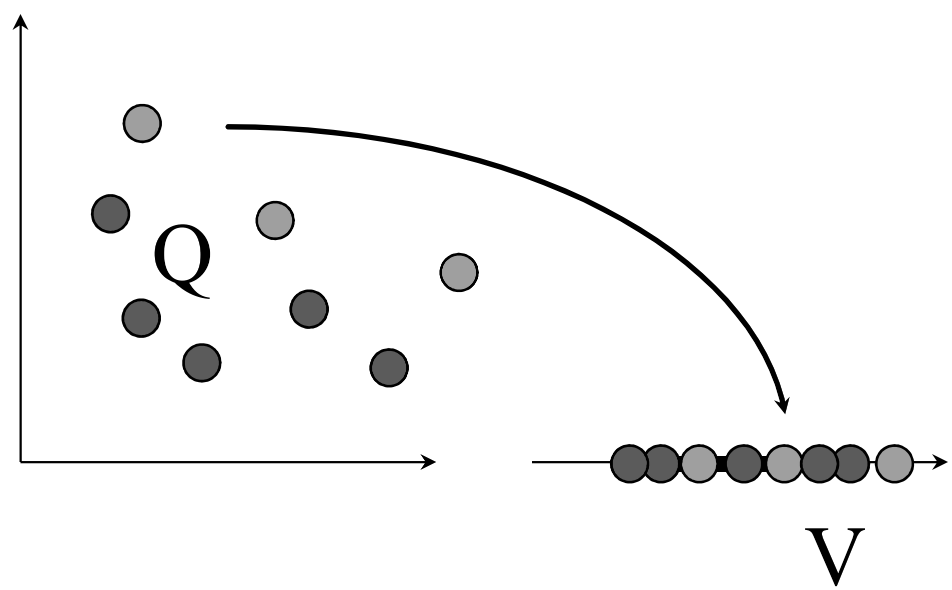


Illustration: An MOLP-Problem

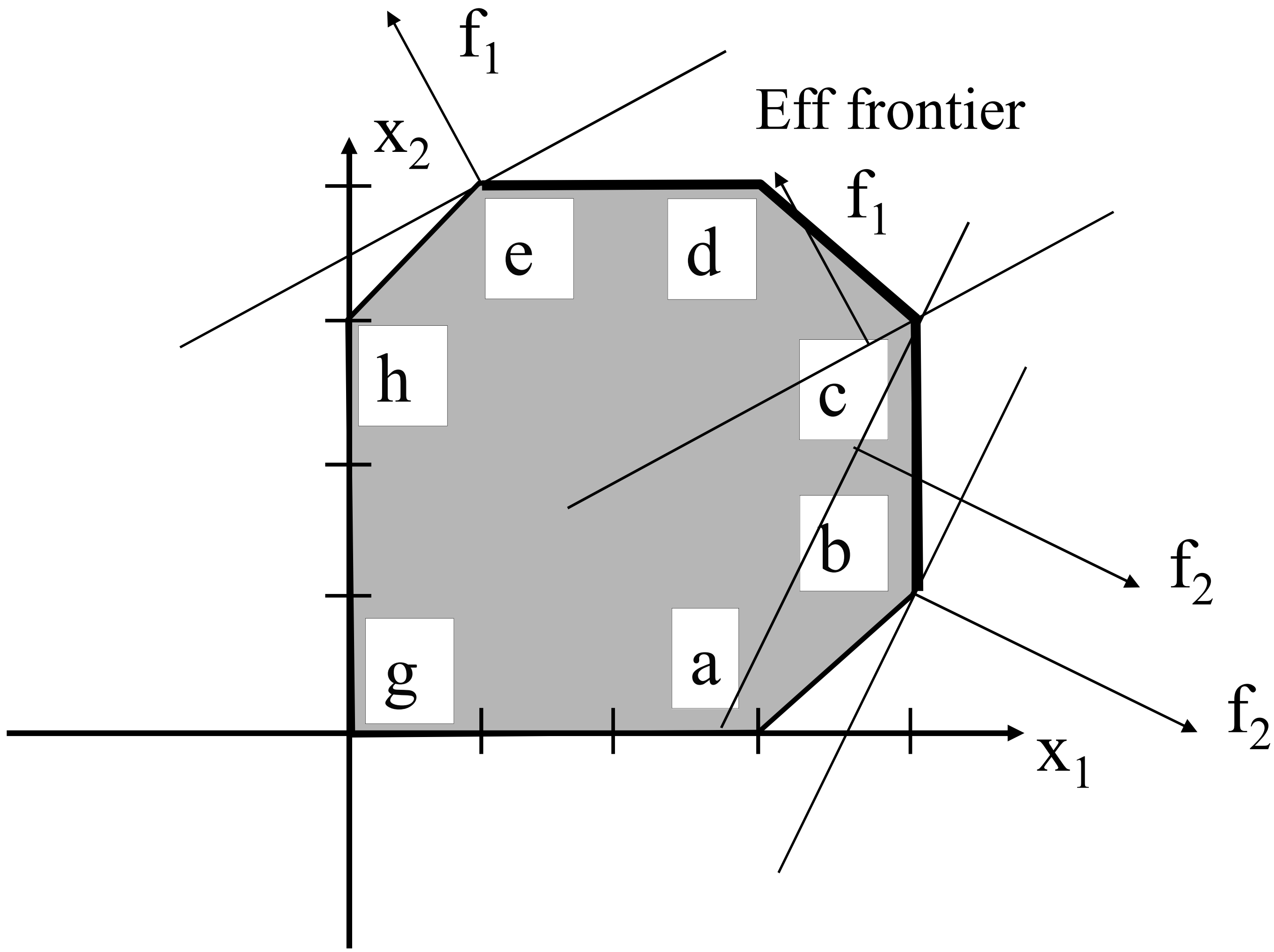
Max $f_1 = -x_1 + 2x_2$

Max $f_2 = 2x_1 - x_2$

Subject to:

$$
\begin{aligned}
x_1 \quad\quad &\leq 4 \\
x_2 &\leq 4 \\
x_1 + x_2 &\leq 7 \\
-x_1 + x_2 &\leq 3 \\
x_1 - x_2 &\leq 3 \\
x_1, x_2 &\geq 0
\end{aligned}
$$

MOLP-Example: In Decision Space



MOLP-Problem: In Criterion Space

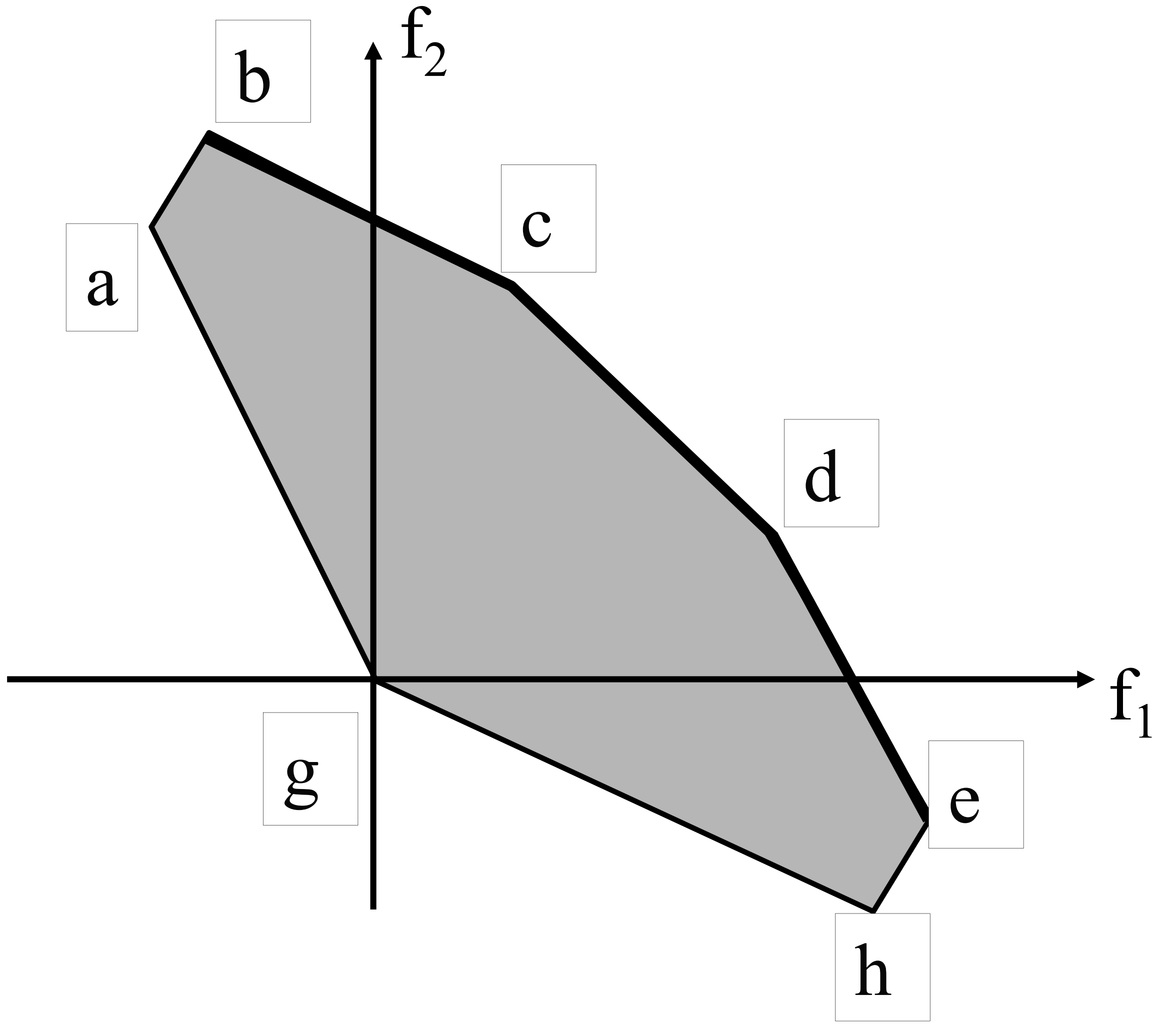
The MOLP-Problem in Criterion Space

$$\max \quad f_1$$
$$\max \quad f_2$$

Subject to:

$$
\begin{aligned}
f_1 + 2f_2 &\le 12 \\
2f_1 + f_2 &\le 12 \\
f_1 + f_2 &\le 7 \\
f_1 - f_2 &\le 9 \\
-f_1 + f_2 &\le 9 \\
f_1 + 2f_2 &\ge 0 \\
2f_1 + f_2 &\ge 0
\end{aligned}
$$

MOLP-Problem: In Criterion Space



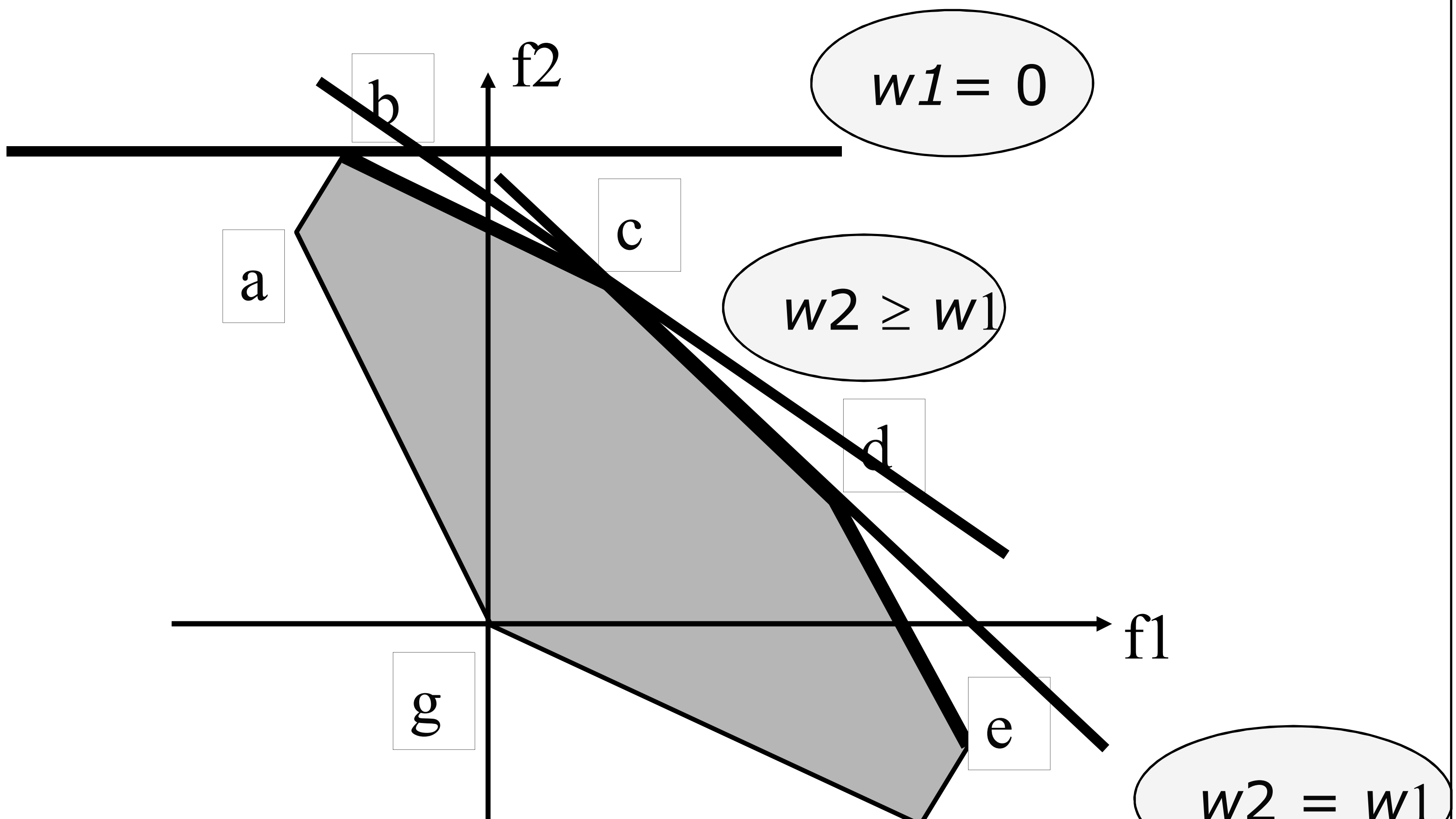
Theorem

$$\begin{array}{ll} Max & \sum w_i z_i(x) \\ st & x \in X \end{array}$$

yields a (supported) efficient solution for any set of weights satisfying $w_i > 0$ i=1,…,p

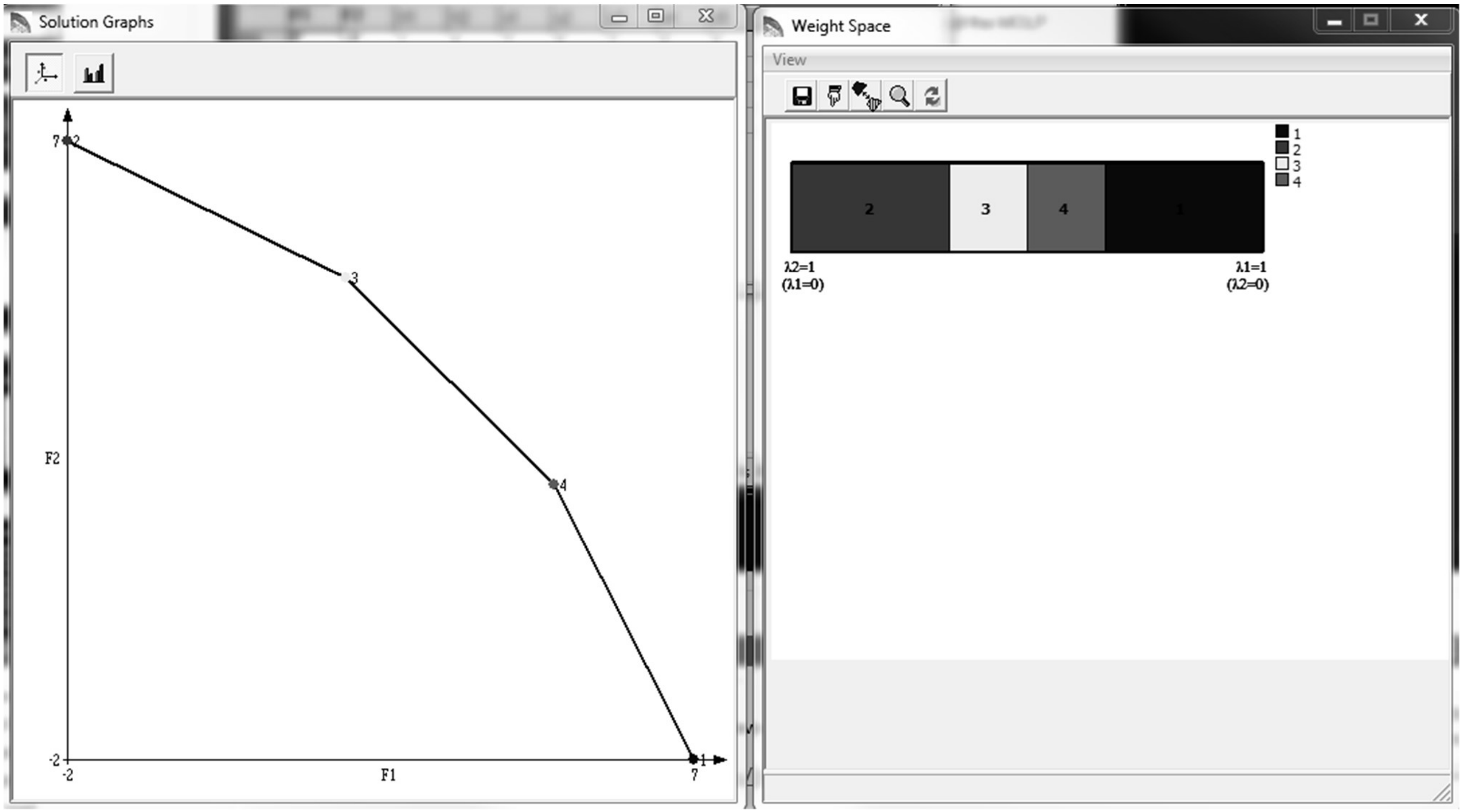
Remark: Changing w_i's systematically, different (supported) efficient solutions can be obtained

Weighted Sums: MOLP



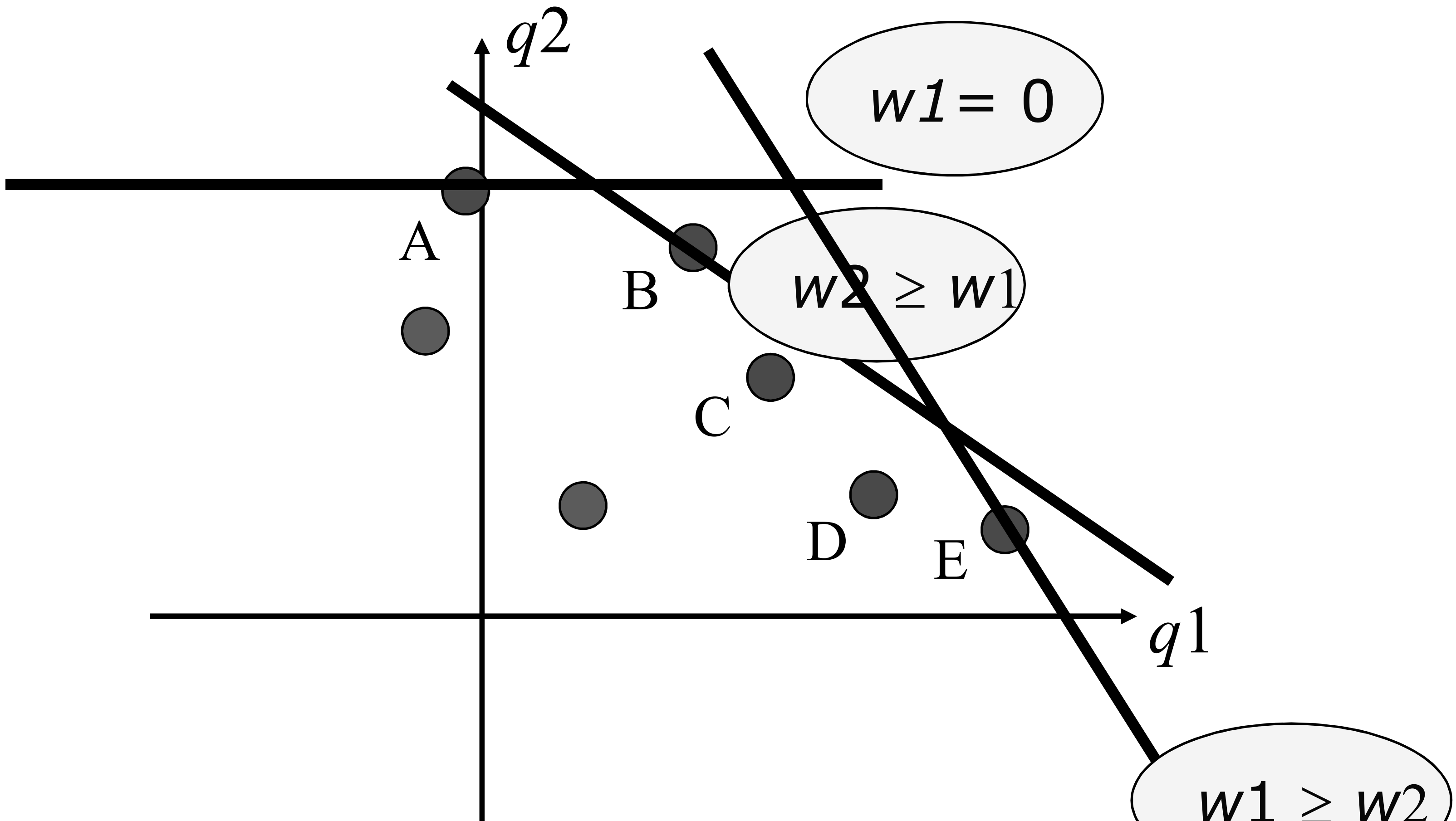
Usually, only extreme supported nondominated solutions are obtained!!

Criteria and weight space

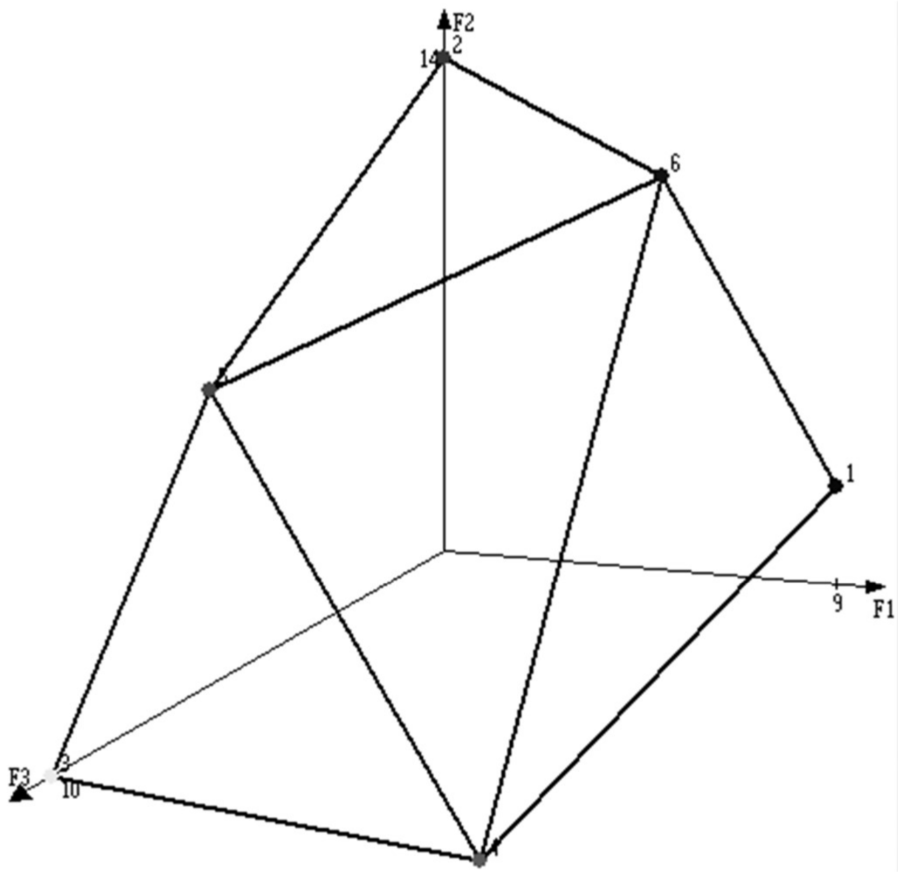


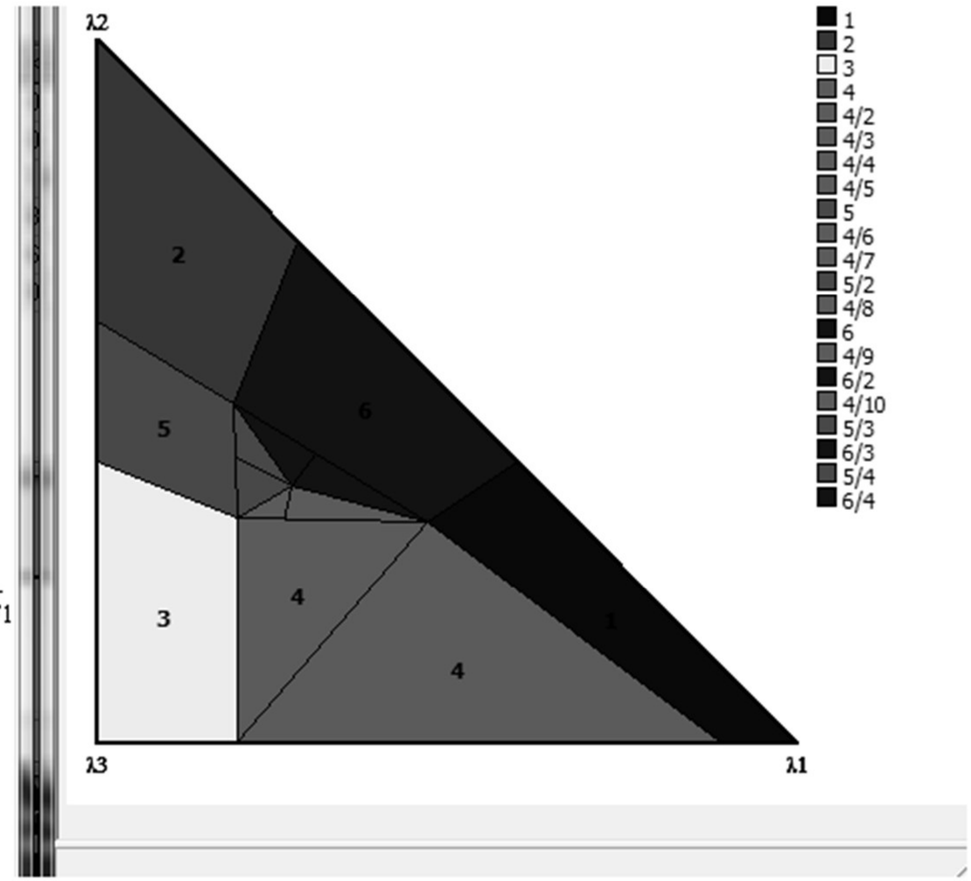
Source: Antunes et al. MOLP software

Weighted Sums: Discrete Alternative Problem



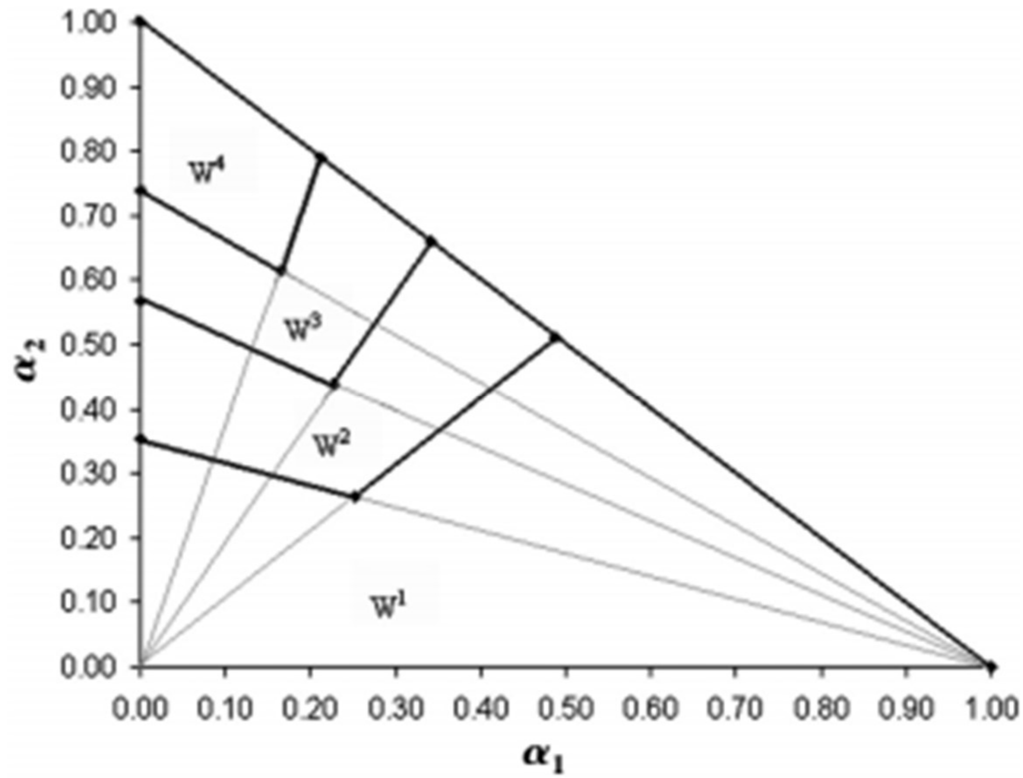
Weight Set - Linear





Source: Antunes et al. MOLP software

Weight Set - Tchebycheff



Source: Bozkurt et al. (2010) Oper. Res.

The ε-constraint approach

- Idea: Write "min. acceptable level" constraints on all but one objective

$$\begin{aligned} &\text{Max } z_k(x) \\ &z_i(x) \geq \varepsilon_i \qquad \forall\, i \neq k \\ &\quad x \in X \end{aligned}$$

does not guarantee an efficient solution

- The solution may be weakly efficient but inefficient

ε-constraint (cont.)

- To guarantee an efficient solution, solve

$$\text{Max } z_k(x) \;\; +\textstyle\sum_{i \neq k} \rho_i z_i(x)$$

$$z_i(x) \geq \varepsilon_i \quad \forall\, i \neq k$$

$$x \in X$$

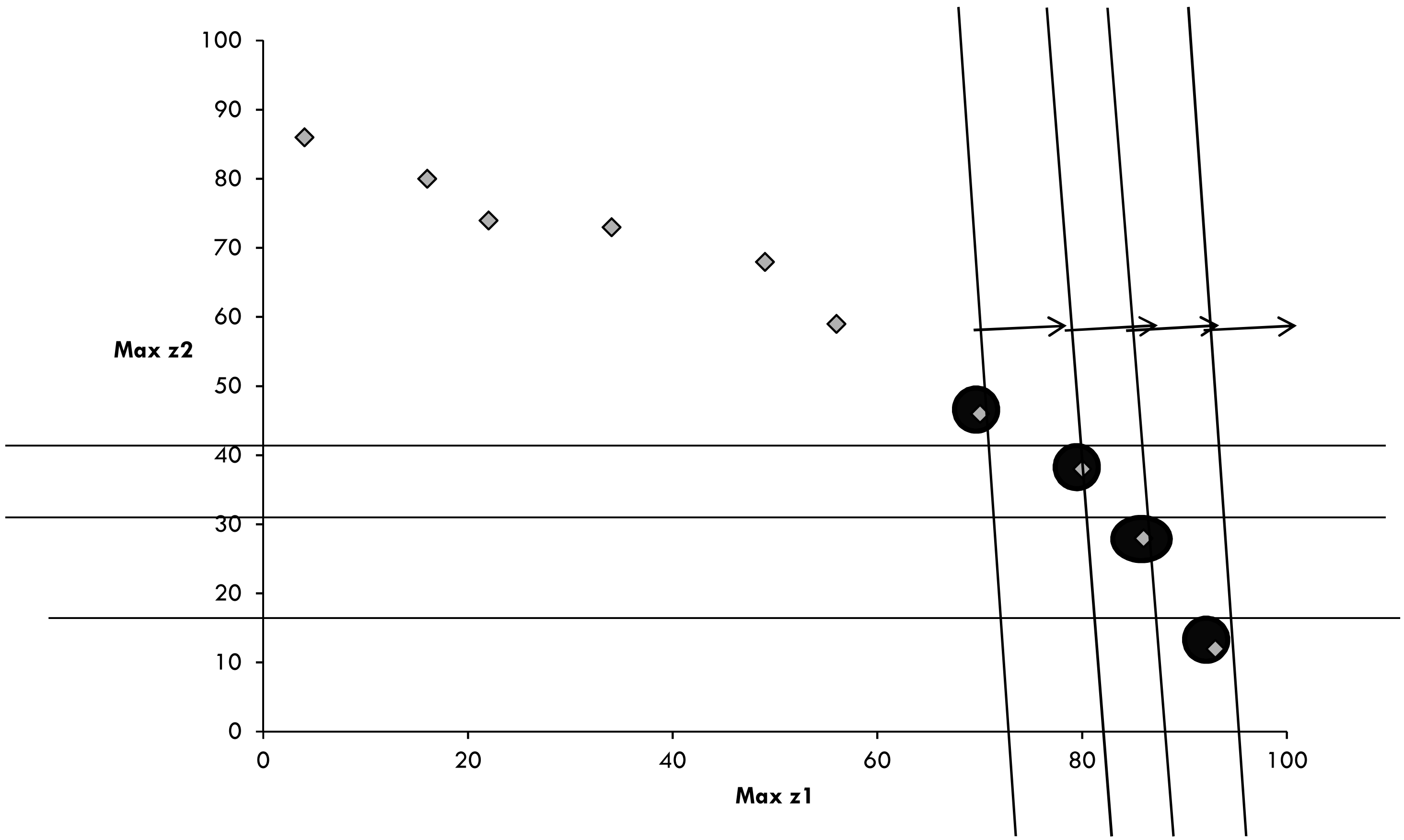
where $\rho_i > 0$ and arbitrarily small

ε-constraint (cont.)

- Changing ε_i values systematically, we can find many (sometimes all) efficient solutions and approximate the efficient frontier
- We can explore different (desirable) parts of the efficient frontier
- In integer programs, we may be able to find all efficient solutions

Example

z_1	z_2
4	86
16	80
22	74
34	73
49	68
56	59
70	46
80	38
86	28
93	12



Choice (Discrete) Problem: Further Outline

- Prior articulation of preferences
  - Estimating a value function
  - Outranking relations
  - AHP
- Progressive articulation of preferences
  - Interactive approaches-implicit value fn.
    - linear
    - quasiconcave
    - general monotone
  - Estimation of the form of value fn.
- Visual Interactive Approaches
  - VIMDA, VISA, …
- Classification and Sorting Problems

Progressive articulation of preferences- Interactive approaches

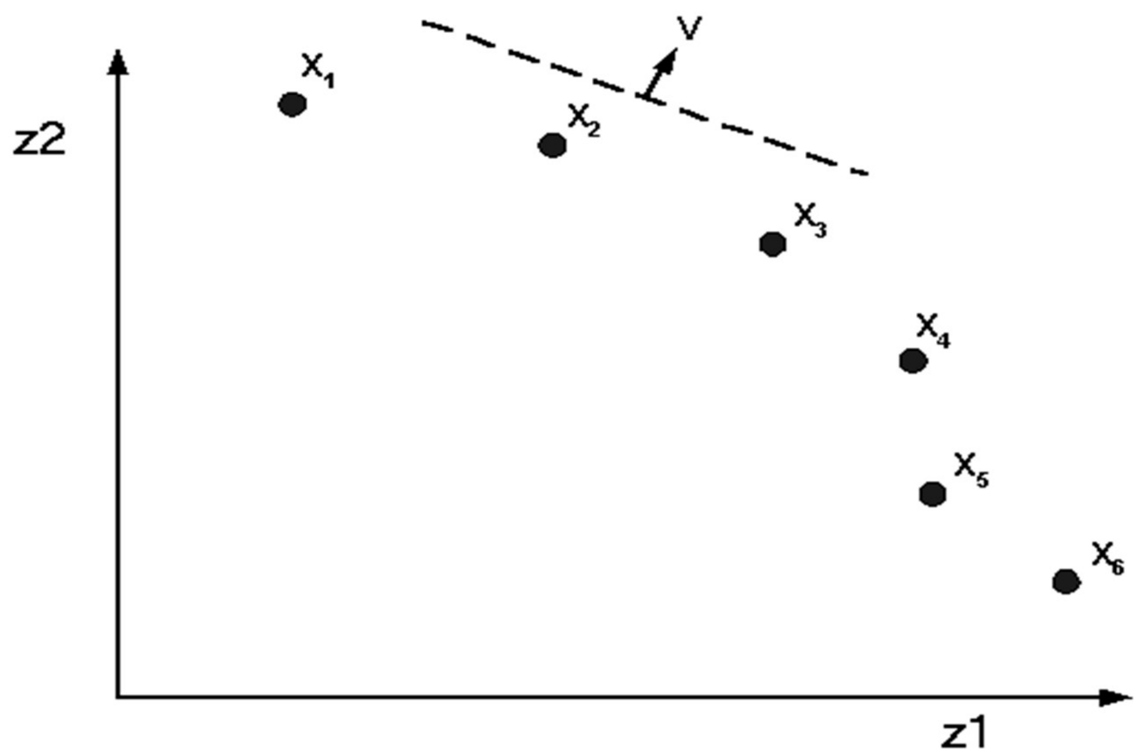
- Suitable for:
  - large # of alts
  - several criteria (say 2-7).
- Aim: converge to “best” alternative quickly.
- Assume
  - the DM can compare pairs of alternatives
  - there exists an implicit value fn, v, consistent with DM’s preferences
  - the general form of v (linear or quasiconcave or general monotone) is known.

Structure of the approach

- The DM compares provided pairs of alternatives.
- Set of alternatives is reduced (based on DM's response and known form of v).
- Repeat until only "best" alternative is left.

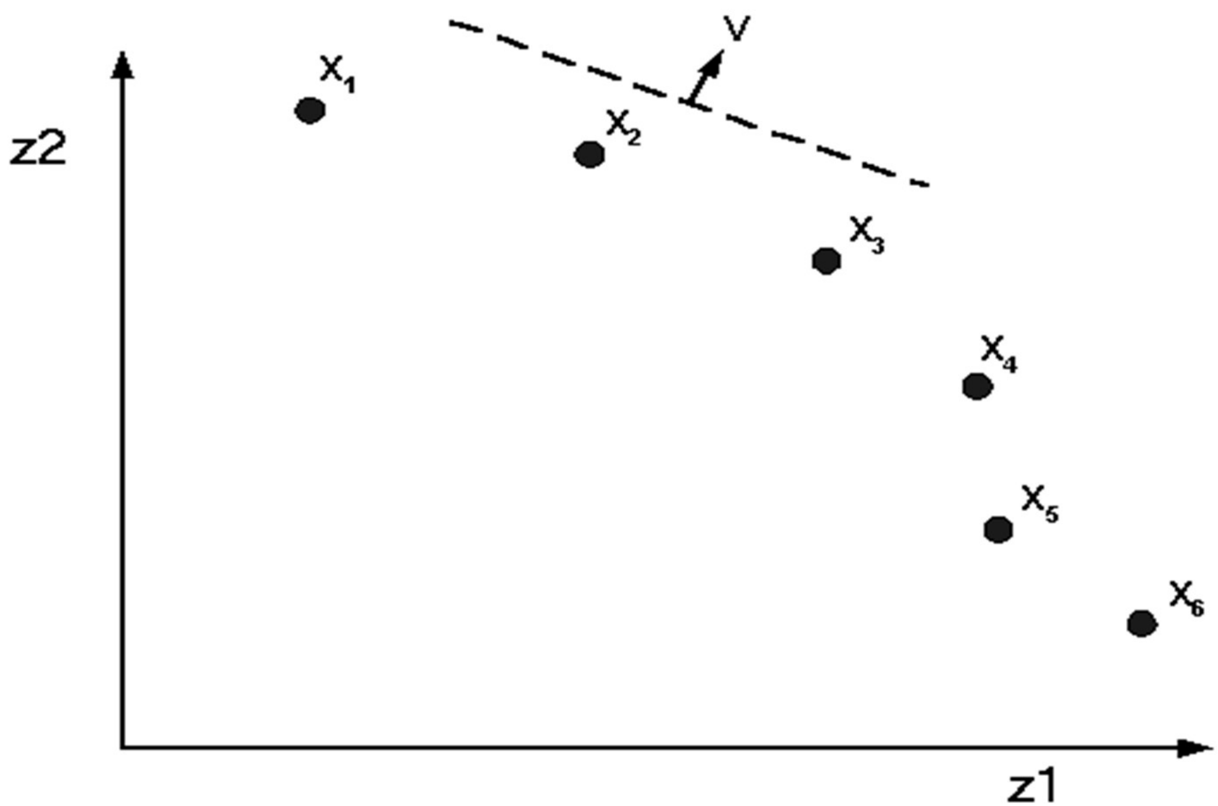
Linear v

- A pair of alts. are adjacent efficient if their convex combinations are not convex dominated



Linear v (Zionts, *EJOR* 1981)

- Theorem: An alt. preferred to all its adj. eff. alts. is "best."



Approach

- Find x* maximizing a linear estimated v
- Ask DM to compare x* with its adjacent efficient solutions
  - If an adjacent eff. solution is preferred update v and start again
- If x* is preferred to all adjacent eff. solutions, Stop.

Stanley Zionts

Quasiconcave v (Korhonen et al, *Mgmt. Sci.* 1984)

- Property of v: decreasing marginal rate of substitution.
- **Thm:** If $v(X_k)=\mathrm{Min}_{i\in S} v(X_i)$ then

  for all Y satisfying

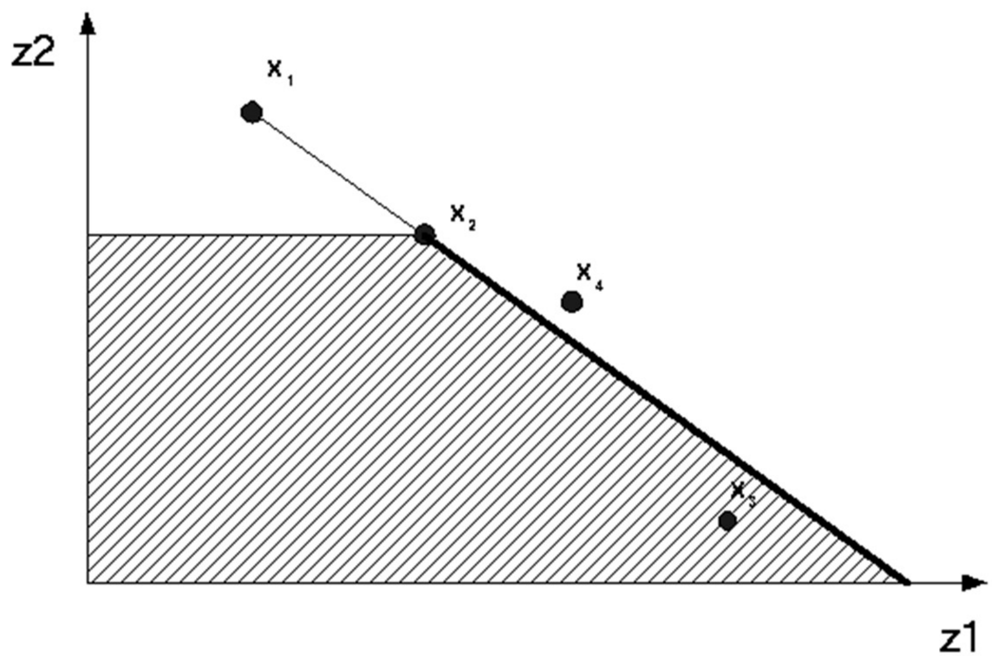
  $C=X_k+\Sigma_{i\in S}\,\mu_i(X_k-X_i)\geq Y \quad \mu_i\geq 0$

we have

$v(X_k)\geq v(Y)$.

Demonstration

Assume $v(X_1) > v(X_2)$



To solve for Y_t

$$(P) \qquad Min\ 0$$
$$st \ \sum_{i \neq k} \mu_i (X_k - X_i) \geq (Y_t - X_k)$$
$$\mu_i \geq 0 \ i \neq k$$

If *(P)* is feasible then $v(X_k) \geq v(Y_t)$

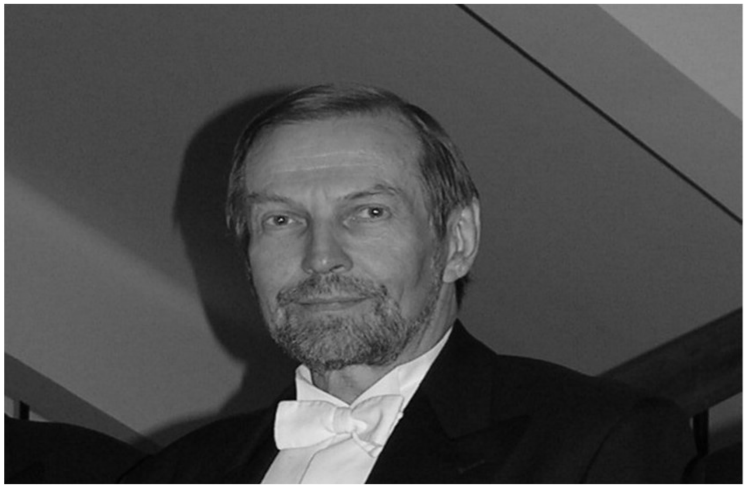
If *(P)* is infeasible then no info.

$$(D) \qquad Max\ \lambda(Y_t - X_k)$$
$$st \ \lambda(X_k - X_i) \leq 0$$
$$\lambda \geq 0$$

- If *(D)* has 0 obj. at optimal: then *(P)* is feasible and $v(X_k) \geq v(Y_t)$
- If *(D)* is positive then *(P)* is infeasible and $v(X_k)$? $v(Y_t)$

Approach

- Ask the DM to compare some pairs of alts.
- Define all cones, and eliminate alts. that fall into any cone, C.
- Continue until a single alt. is left.



Pekka Korhonen

Jyrki Wallenius

Variations

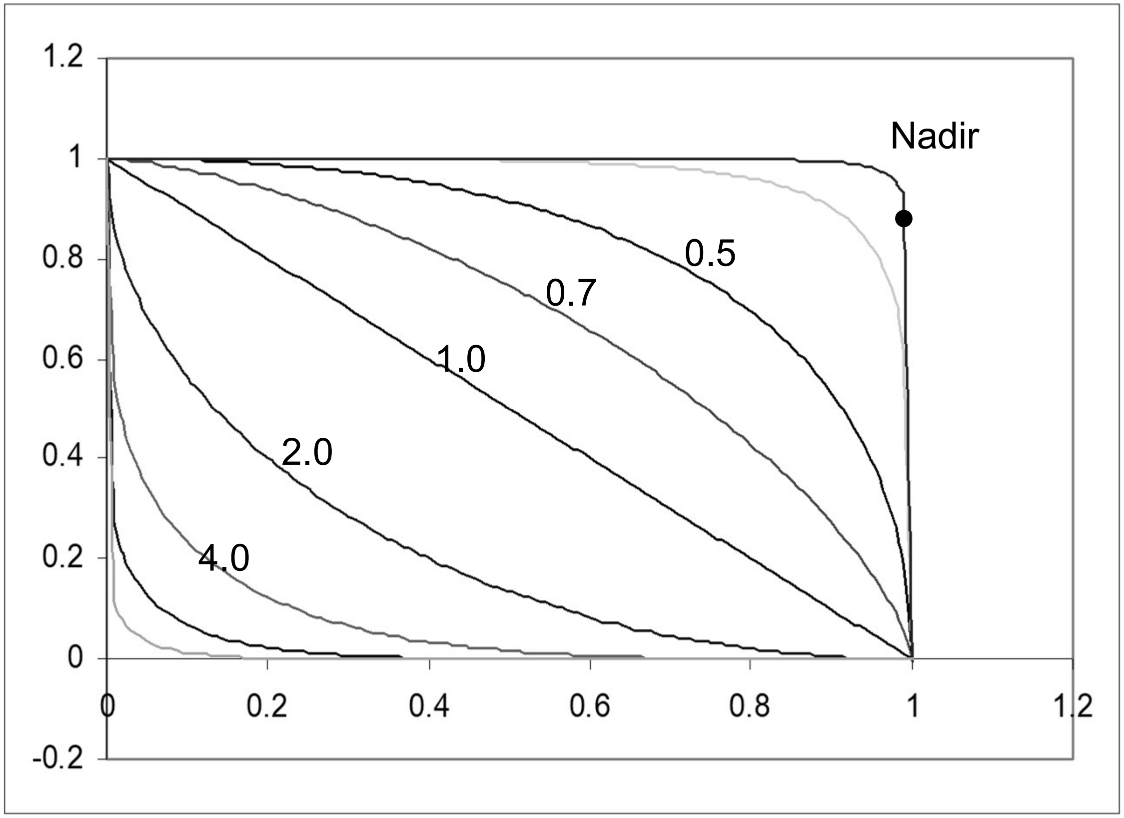
- Köksalan, Karwan and Zionts (*IEEE SMC* 1984) and Köksalan and Taner (*EJOR* 1992) make modifications to improve the convergence

v : L_α^w (Karakaya et al., EJOR 2018)

$$L_\alpha^{\mathbf{w}}(|\mathbf{q}-\mathbf{r}|) = \begin{cases} \left(\sum_{j=1}^{p}\left(w_j|q_j-r_j|\right)^\alpha\right)^{1/\alpha}, & \text{if } 1 \le \alpha < \infty, \\ \max\limits_{j=1,\dots,p}\{w_j|q_j-r_j|\}, & \text{if } \alpha = \infty, \end{cases}$$

- Property of v:
  - approximates quasiconcave/quasiconvex functions well
  - takes a variety of forms based on α and **w**

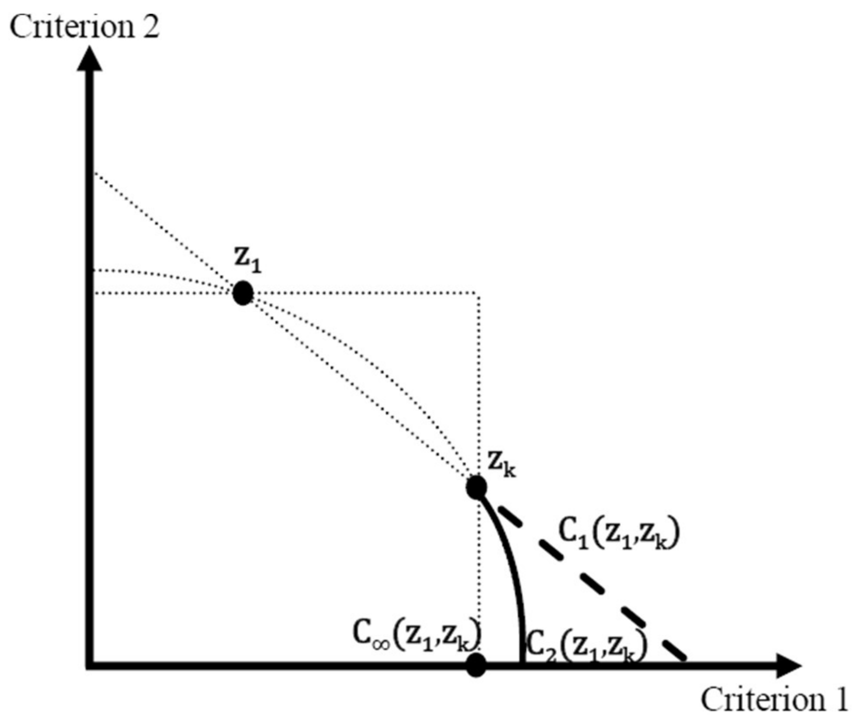
$(1-u')^{\alpha} + (1-v')^{\alpha} = 1 \quad \alpha > 0$



We consider weighted version and $\alpha \geq 1$

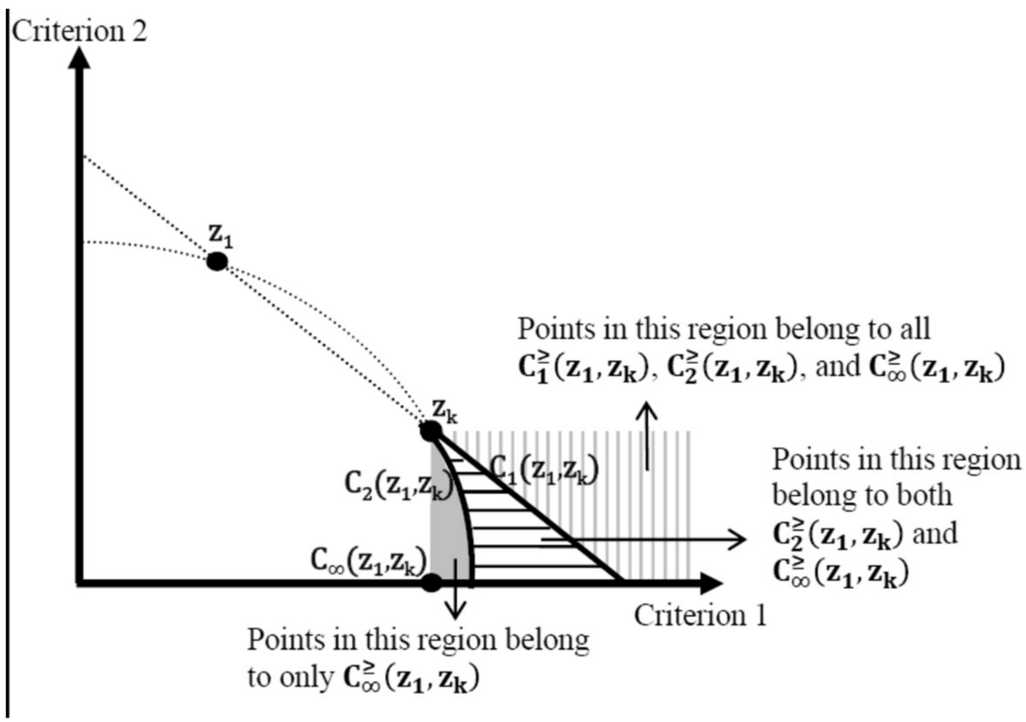
Inferior Alternatives

Assume that z_1 is preferred to z_k, i.e. $v(z_1)<v(z_k)$



Inferior Alternatives

Assume that z_1 is preferred to z_k, i.e. $v(z_1)<v(z_k)$



Approach

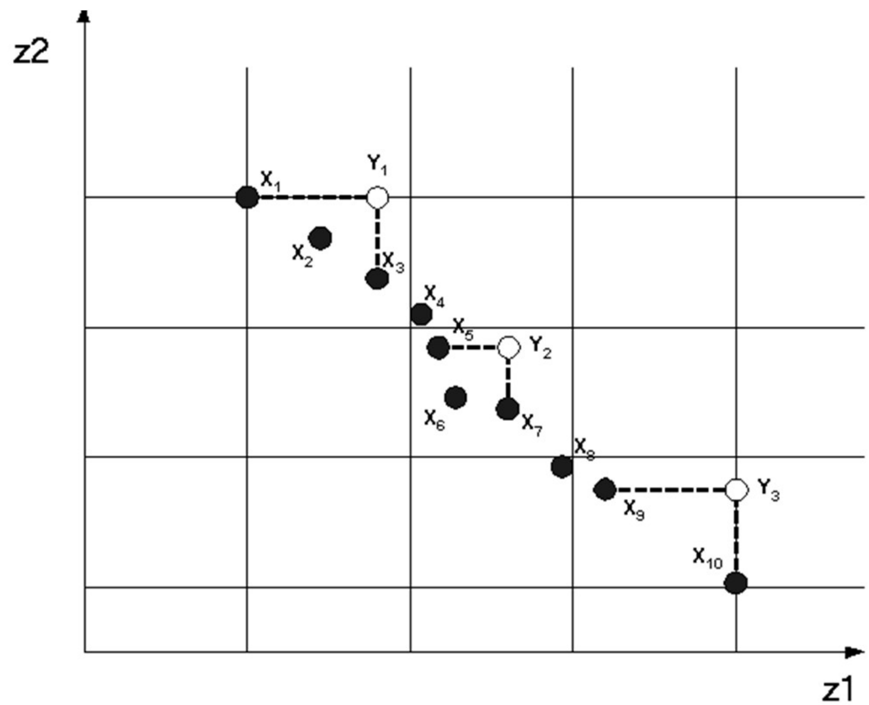
- Start with linear v.
- Ask the DM to compare some pairs of alts.
- Eliminate inferior alts. that fall into any $C_\alpha(z_i,z_k)$
- Update parameters of v. If necessary, increase α
- Continue until a single alt. is left

- If α is known, continue until an alt. is preferred to all its "α-adjacent" alts.

General monotone v
(Köksalan and Sagala 1995, *Man Sci*)

- Assume only “more is better”
- Eliminate only based on dominance

Demonstration



Approach

- Group alts into partitions & find PIs
- If DM prefers an alt to a PI, delete whole partition
- If necessary, reduce group sizes and redefine PIs
- Repeat until a single alt is left

Estimate form of v

- Korhonen et al (1986, 1993), Salminen et al (1989)
  - Solve various LPs to test violations of linearity and quasiconcavity
- Köksalan & Sagala (1995)
  - Use convex combinations of alts to quickly identify violations of linearity and quasiconcavity
- A General Approach
  - Estimate form of v
  - Use most efficient algorithm available for identified form

Visual Interactive Approaches

- VIMDA (Korhonen ‘88 *EJOR*)
- AIM (Lotfi et. al. ’92 *C&OR*)
- …

Classification and Sorting

- Place alternatives in different classes (based on similarity-classification) or preference-ordered classes (sorting)
- Examples
  - Patients into disease groups based on symptoms
  - Country risk assessment, credit risk assessment
  - Selecting applicants for different scholarships/graduate programs
  - Selecting projects for different kinds of funding policies

An Approach
(Köksalan and Ulu, *EJOR* 2003)

- Linear Utility Function-not explicitly known

$$u(X_i) = \sum_j \lambda_j x_{ij}$$

Preference classes are well defined

$X_i \in C \Leftrightarrow LB(C) \leq u(X_i) \leq UB(C)$ (unknown to us)

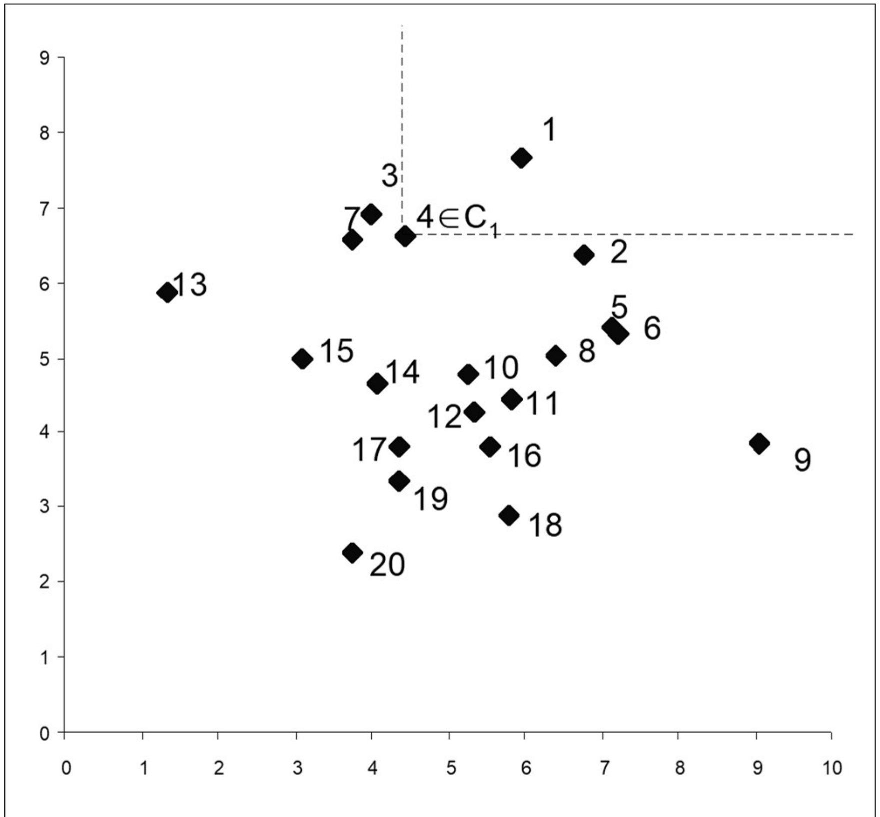
- DM can correctly place alternatives in preference classes consistent with his/her underlying utility function and the bounds

Algorithm

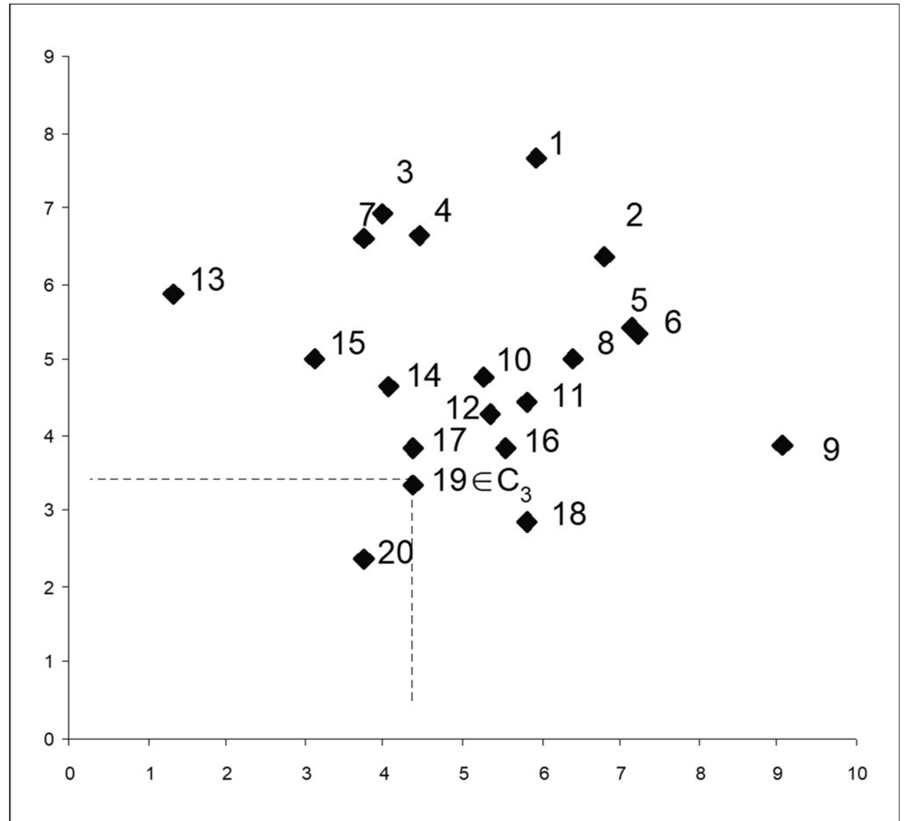
- Ask DM to place alternatives in a preference class; C_i
- To place alternatives in implied preference classes, C_i', use:

  - Dominance
  - Convex combinations
  - Weight space reduction

Dominance

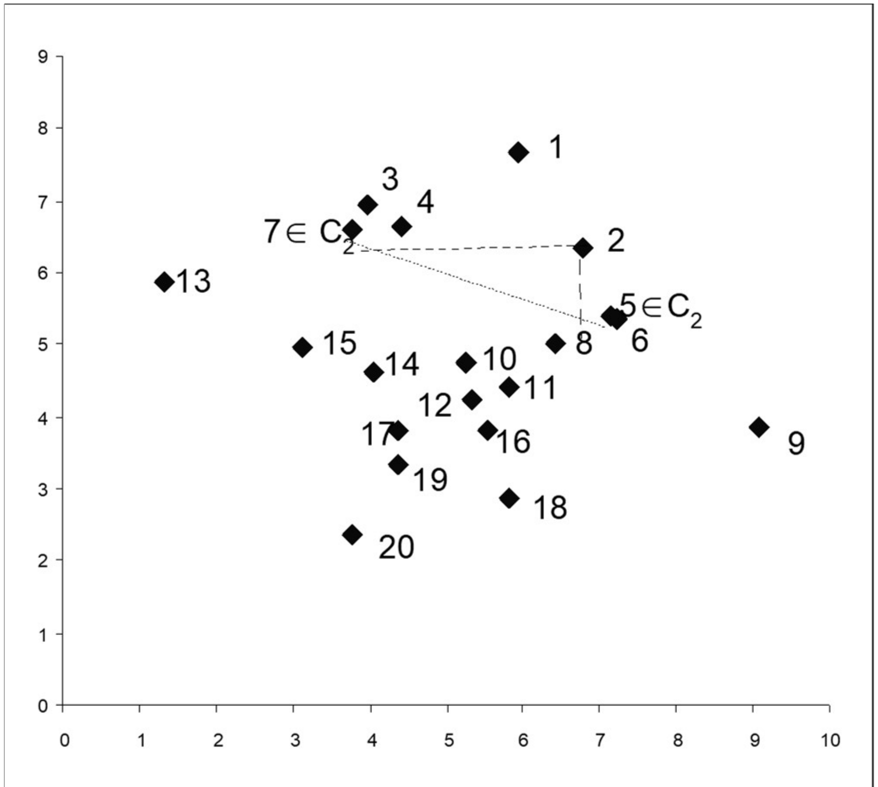


Placing an alternative in C_1 utilizing dominance

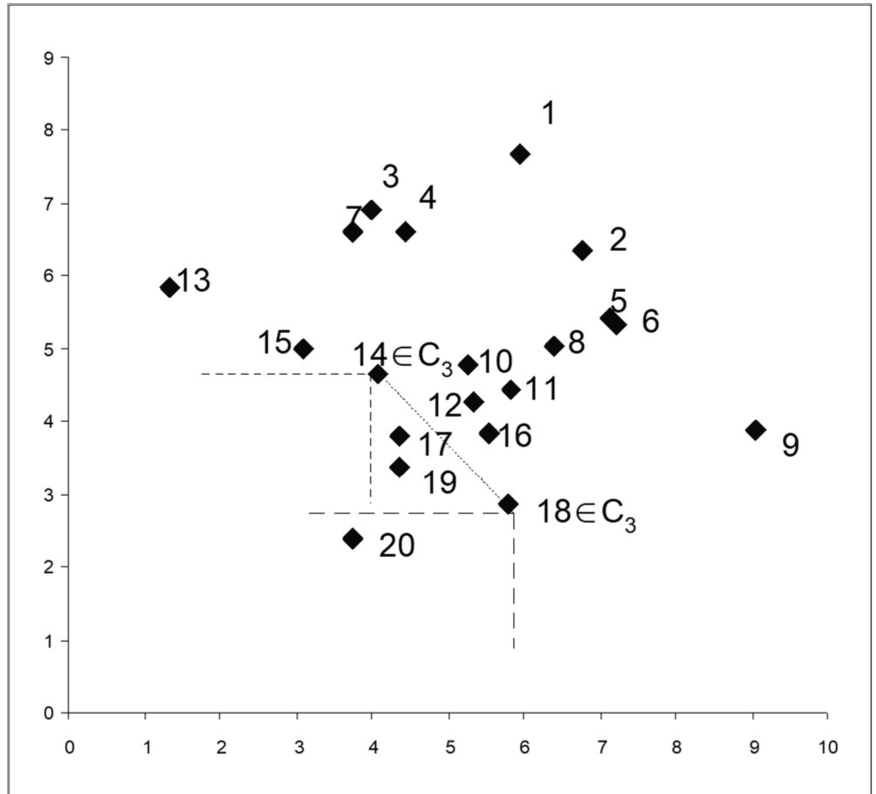


Placing an alternative in C_3 utilizing dominance

Convex Combinations

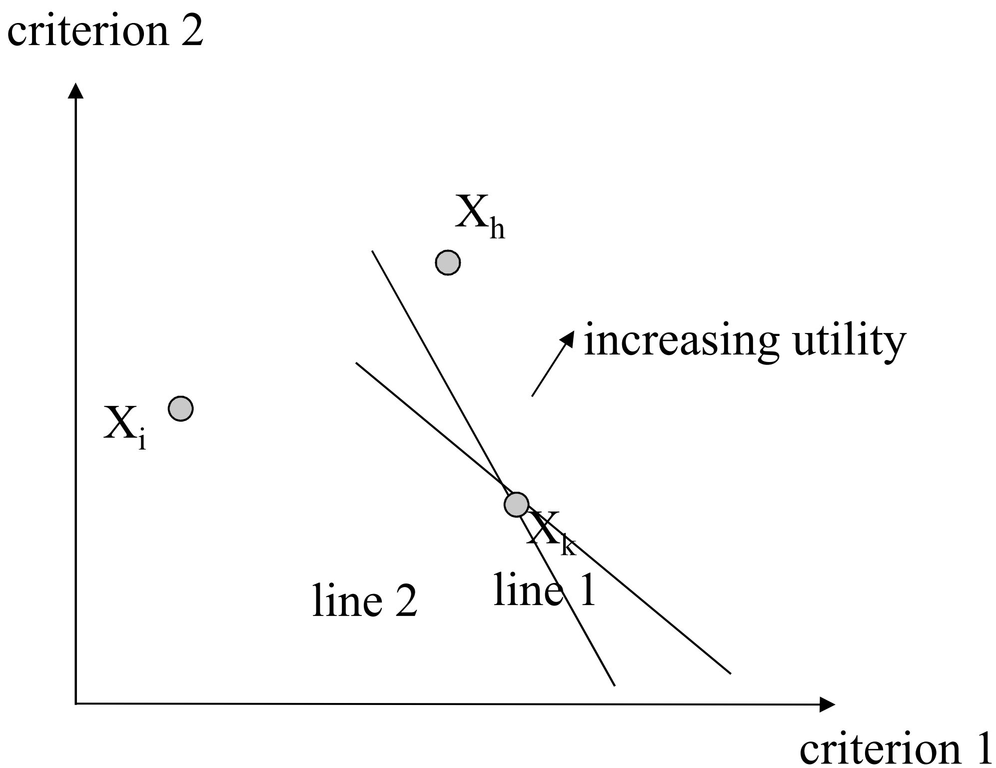


Using convex dominance to detect the worst possible class of an alternative



Using convex dominance to detect the best possible class of an alternative

Weight Space Reduction



Variations

- An Interactive sorting method for an additive v (Koksalan and Ozpeynirci, *COR* 2009)
- An Interactive probabilistic sorting method for an additive v (Bugdaci, Koksalan, Ozpeynirci, Serin, *IIE Transactions* 2013)
- An Interactive sorting method for quasiconcave v (Ulu & Koksalan, NRL 2014)

Design (Continuous Solution Space) Problem

•Steuer, Multiple Criteria Optimization, Wiley 1986

•Miettinen, Nonlinear Multiobjective Opt, Kluwer 1999

Further Outline

- Prior articulation of preferences
  - GP
- Progressive articulation of preferences
  - Interactive approaches-implicit value fn.
- Posterior articulation of preferences

Progressive Articulation of Preferences

- Geoffrion, Dyer, and Feinberg (GDF) ’72 *Man. Sci.*
- Benayoun et al. (STEM) ’71 *Math. Prog.*
- Zionts & Wallenius (Z-W) ’76 *Man. Sci.*
- Steuer & Choo ’83 *Math. Prog.*
- Köksalan & Karasakal ’06 *JORS*
- Miettinen et al. ’10 *EJOR*

* Visual Aids

- Korhonen & Laakso ’86 *EJOR*
- Korhonen & Wallenius ’88 *NRL*

and many others

Frank-Wolfe (F-W) NLP (known v)

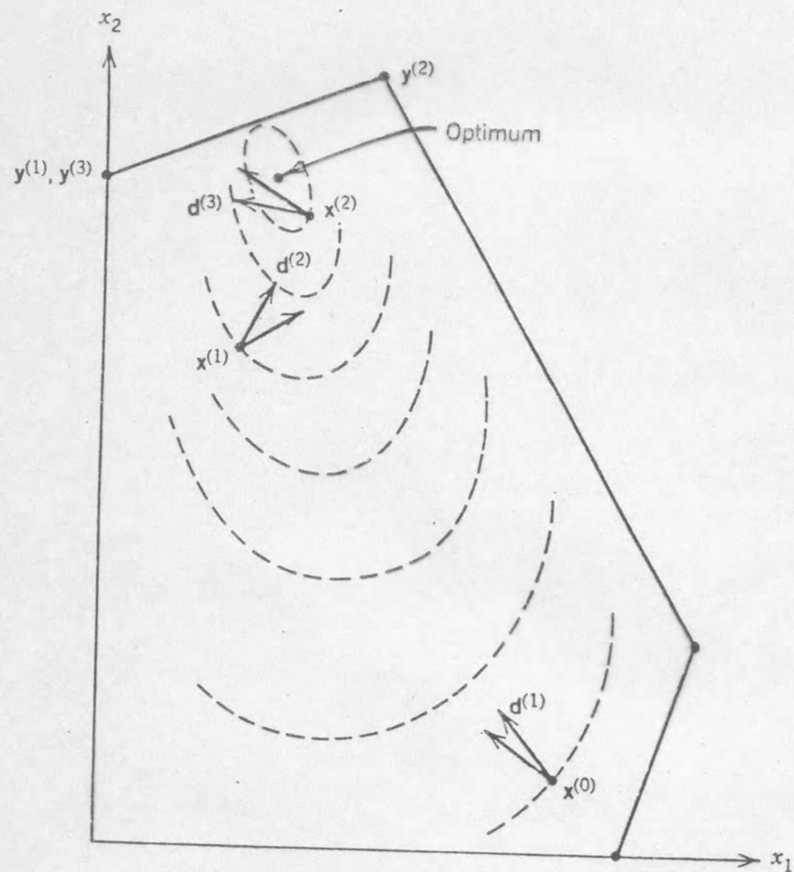


Figure 13.6. Graph of Example 4.

Source:
Steuer (1986)

Example

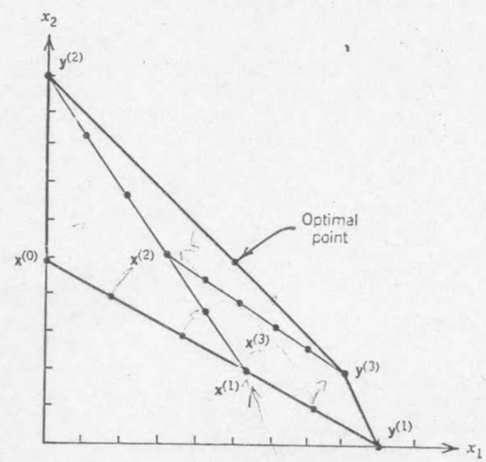


Figure 13.10. Graph of GDF example.

TABLE 13.1
Iteration Data for the GDF Example

	$h = 1$		$h = 2$		$h = 3$	
$\mathbf{x}^{(h-1)}$	(0, 5)		(5.4, 2)		(3.24, 5.2)	
$\mathbf{g}$	(72, 12)		(115.2, 245.8)		(326, 223)	
$\mathbf{y}^{(h)}$	(9, 0)		(0, 10)		(8, 2)	
$\mathbf{d}^{(h)}$	(9, −5)		(−5.4, 8)		(4.76, −3.2)	
	Stepwise Criterion Vectors	*U*	*Stepwise Criterion Vectors*	*U*	*Stepwise Criterion Vectors*	*U*
	(0, 5)	36.0	(5.4, 2)	368.6	(3.24, 5.2)	691.1
	(1.8, 4)	196.0	(4.32, 3.6)	598.9	(4.192, 4.56)	833.3
	(3.6, 3)	338.6	(3.24, 5.2)	691.1	(5.144, 3.92)	913.8
	(5.4, 2)	368.6	(2.16, 6.8)	607.5	(6.096, 3.28)	922.4
	(7.2, 1)	269.0	(1.08, 3.4)	382.3	(7.048, 2.64)	858.2
	(9, 0)	100.0	(0, 10)	121.0	(8, 2)	729.0
$\mathbf{z}^{(h)}$	(5.4, 2)		(3.24, 5.2)		(6.096, 3.28)	
$U(\mathbf{z}^{(h)})$	368.6		691.1		922.4	

GDF (*Man. Sci.* 1972)

- There are two problems when v is not known:
  1. We don't know the gradient (steepest ascent direction)
  2. We cannot find the best point along that direction

GDF (cont.)

To obtain preference info:

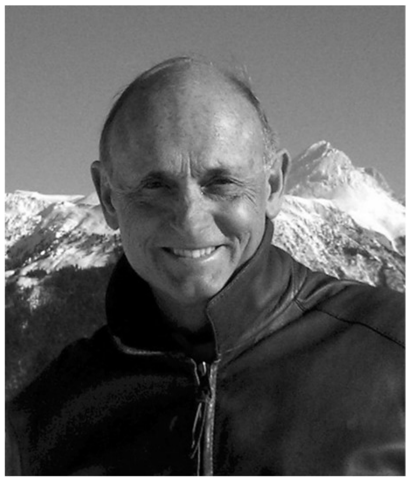
1. Locally approximate v by

   $$F(x) = \sum w_i z_i(x)$$

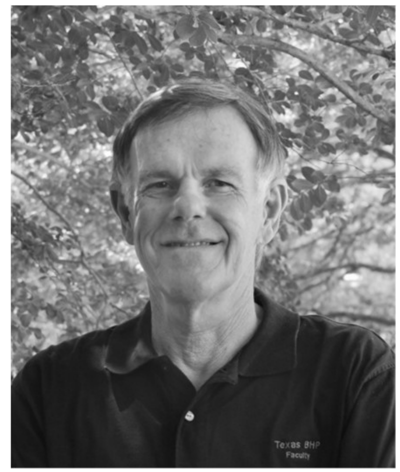
   Let w_1=1. Estimate w_i asking the DM local tradeoffs between z_1 and z_i.

   Find gradient of $F(x)$ at the current x.

2. Ask the DM the best of several discrete points along the direction



Art Geoffrion



Jim Dyer

The Step Method (STEM) Benayoun, deMontgolfier, Tergny, and Larichev (*Math. Prog.* 1971)

Oleg Larichev

In each iteration, solve (P):

$$
\begin{aligned}
&\boldsymbol{Min}\,\alpha \\
&\boldsymbol{st}\quad \alpha \ge \lambda_i (z_i^* - z_i(\boldsymbol{x})) \quad \boldsymbol{i} \notin \boldsymbol{J}^* \\
&\qquad z_i(\boldsymbol{x}) \ge z_i(\boldsymbol{x}^{\boldsymbol{current}}) - \Delta_i \qquad \boldsymbol{i} \in \boldsymbol{J}^* \\
&\qquad\qquad \boldsymbol{x} \in \boldsymbol{X}
\end{aligned}
$$

Minimizes a weighted Tchebycheff distance from the ideal point

Parameters

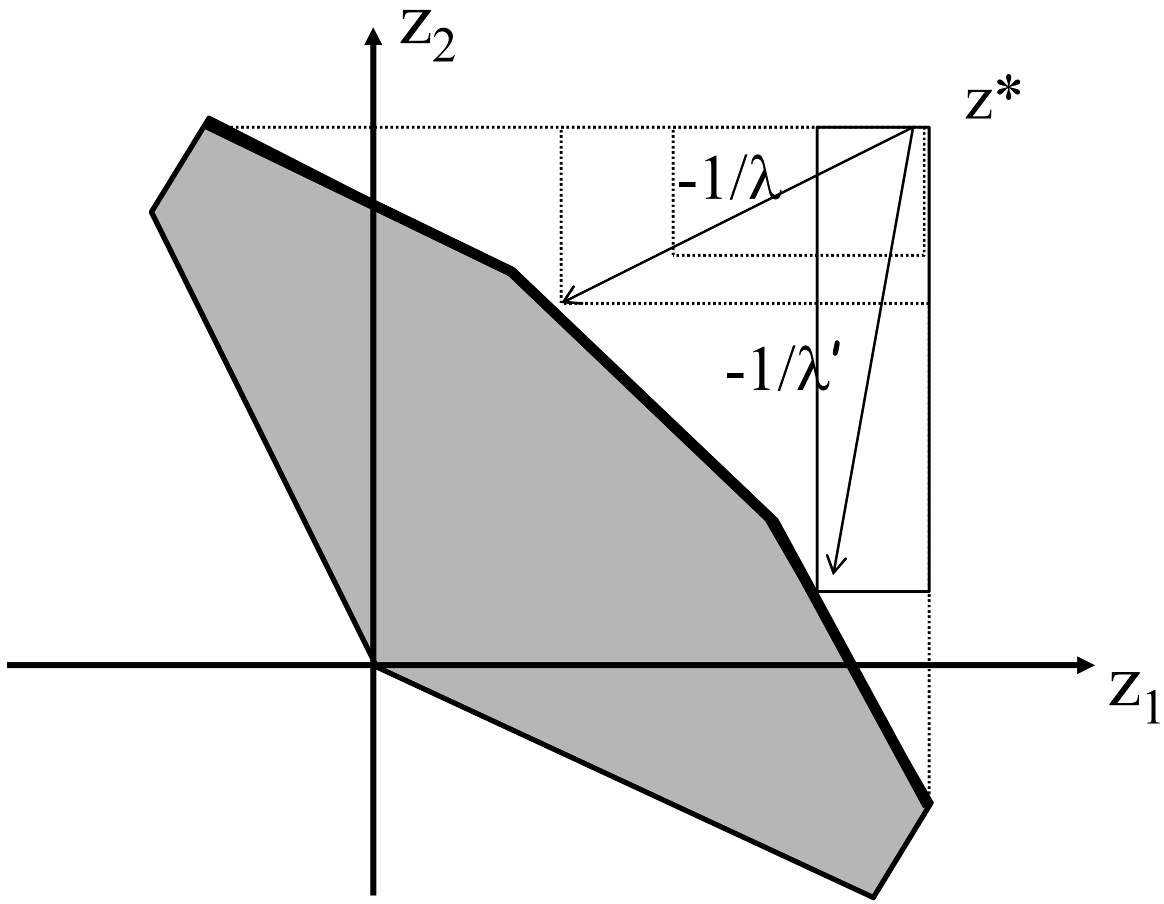
λ_i : a weight for obj. i

Δ_i: amount DM is willing to sacrifice in obj. i

z^*: is the ideal point

J^*: Set of objectives DM is willing to sacrifice from

In Criterion Space



Z-W Method (*Man. Sci.* 1976)

- Assume
  - v is linear
  - z_i is linear for all i
  - X is a polytope

Z-W (cont.)

1. Find an efficient extreme point solution maximizing an estimated linear v
2. Find its adjacent efficient solutions
3. Ask the DM if he/she likes tradeoffs towards any adjacent eff solutions
   - If not; stop.
   - If yes; update the estimated v and go to 1.

Augmented w. Tchebycheff function

Source: Steuer (1986)

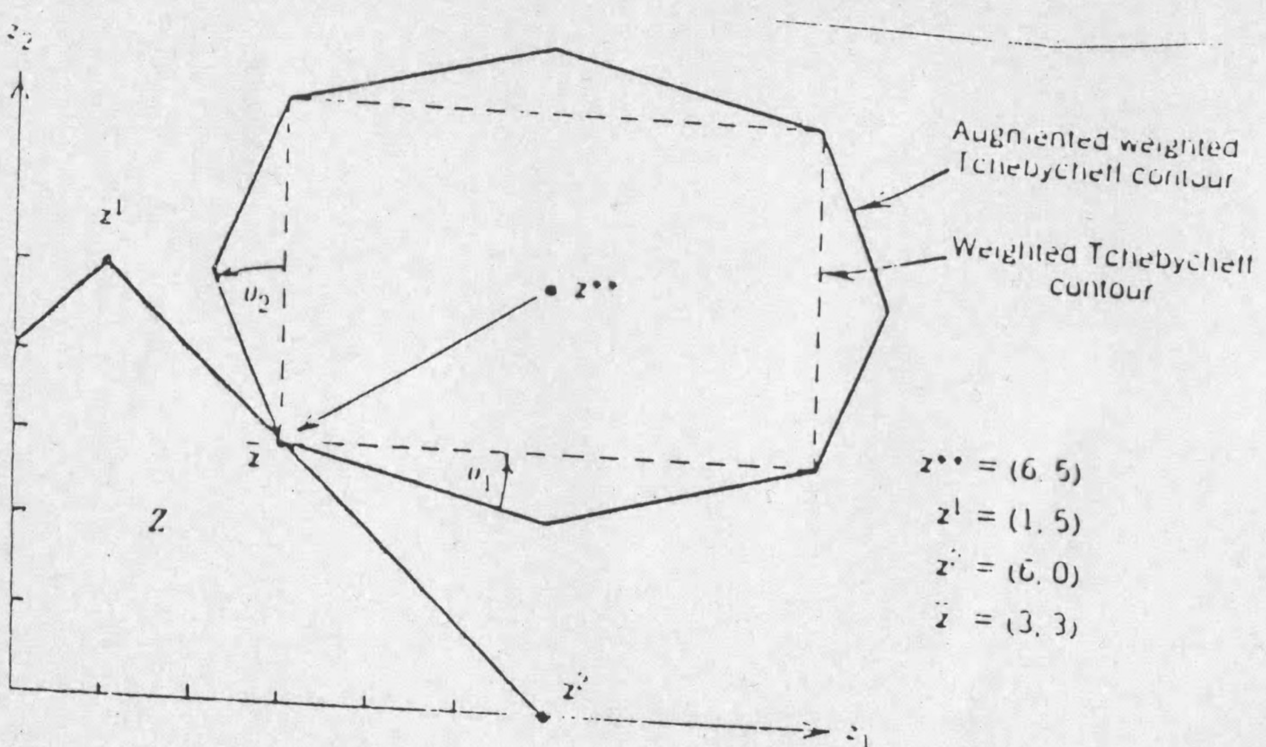


Figure 14.4. Contours of the ... metric intersecting Z

Augmented w. Tchebycheff program

$$
\text{(TP)} \qquad \boldsymbol{Min}\ \alpha + \rho \sum (z_i^* - z_i(\boldsymbol{x}))
$$

$$
\alpha \geq \lambda_i (z_i^* - z_i(\boldsymbol{x}))
$$

$$
\boldsymbol{x} \in \boldsymbol{X}
$$

Examples

428 INTERACTIVE WEIGHTED TCHEBYCHEFF PROCEDURE

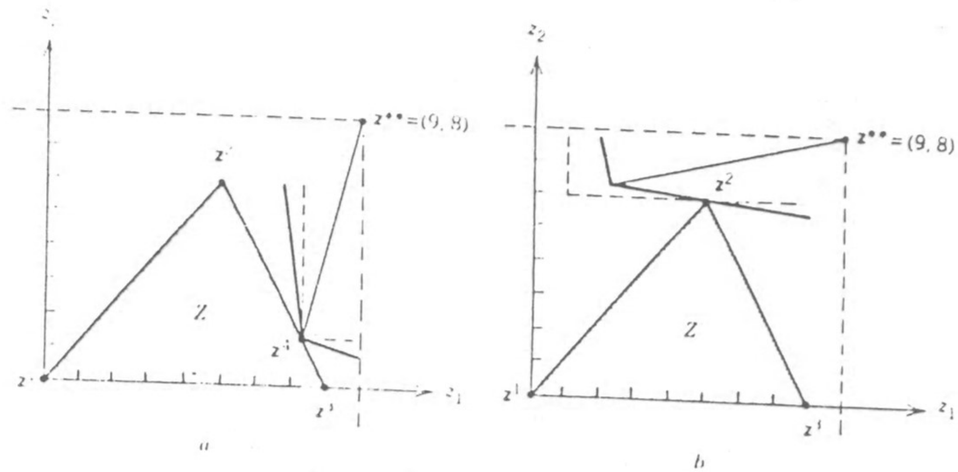


Figure 14.7. Graphs of Example 6.

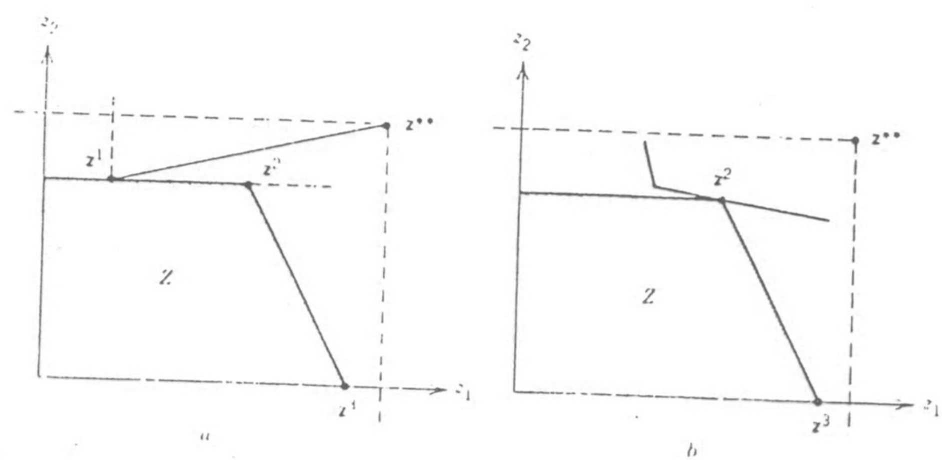
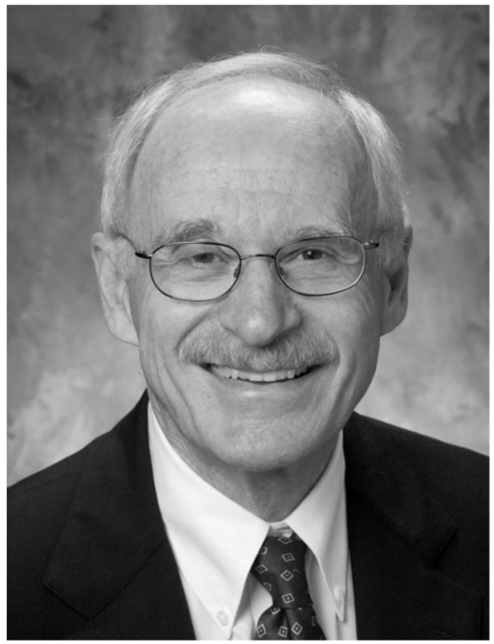


Figure 14.8. Graphs of Example 7.

Source: Steuer (1986)

Interactive w. Tch. Approach
Steuer and Choo (*Math. Prog.* 1983)

- Randomly generate weights
- With each weight set solve (TP) to find a set of eff solns
- Ask the DM best of a representative (small) set of eff solns
- Shrink the weight set around the weights favoring chosen solution
- Repeat several iterations



Ralph Steuer

Achievement Scalarizing Program (ASP)

$$\mathit{Minimize} \;\; \alpha - \varepsilon \sum_{i=1}^{m} z_i(x)$$

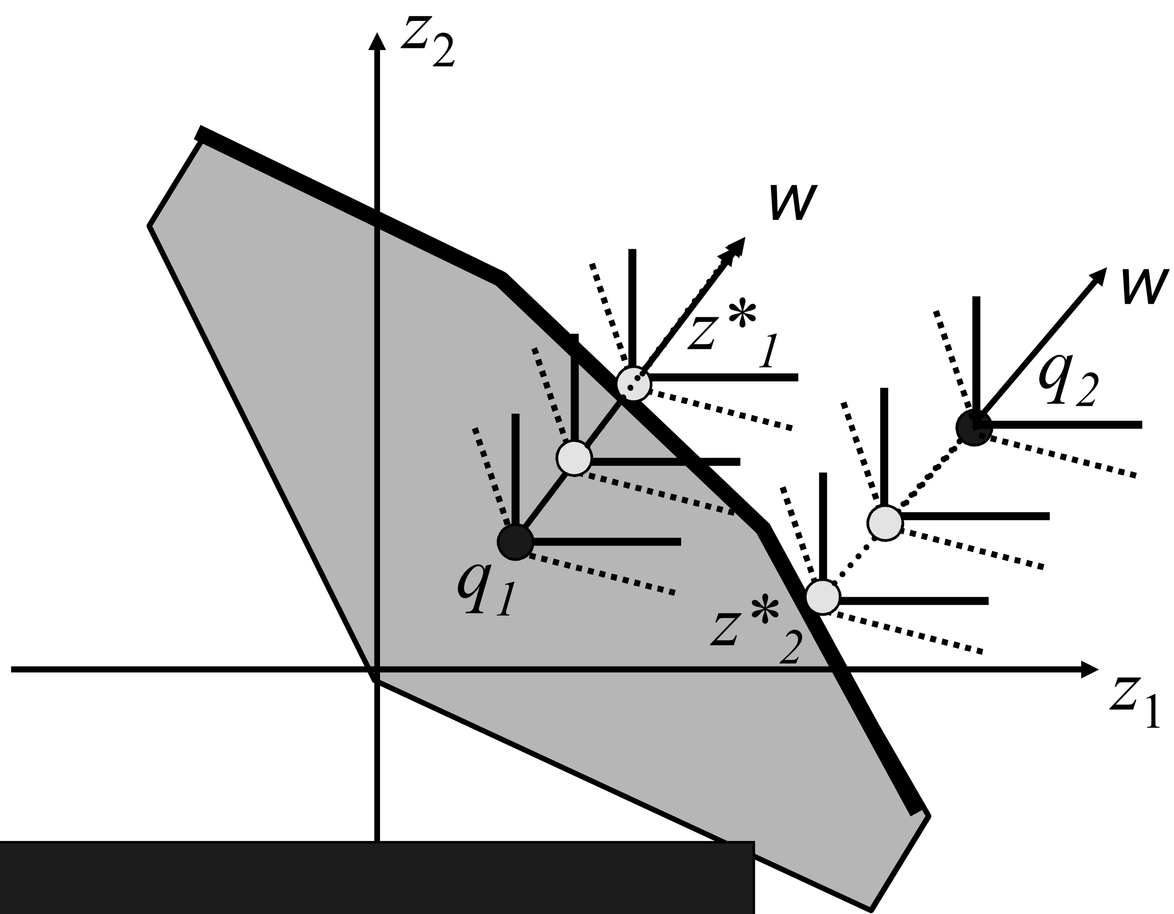
$$\mathit{subject\,to:} \;\; \alpha \geq \lambda_i \left(q_i - z_i(x)\right) \; \forall\, i$$

$$x \in X$$

ASP projects q onto the efficient frontier in the direction 1/λ (or –1/λ).

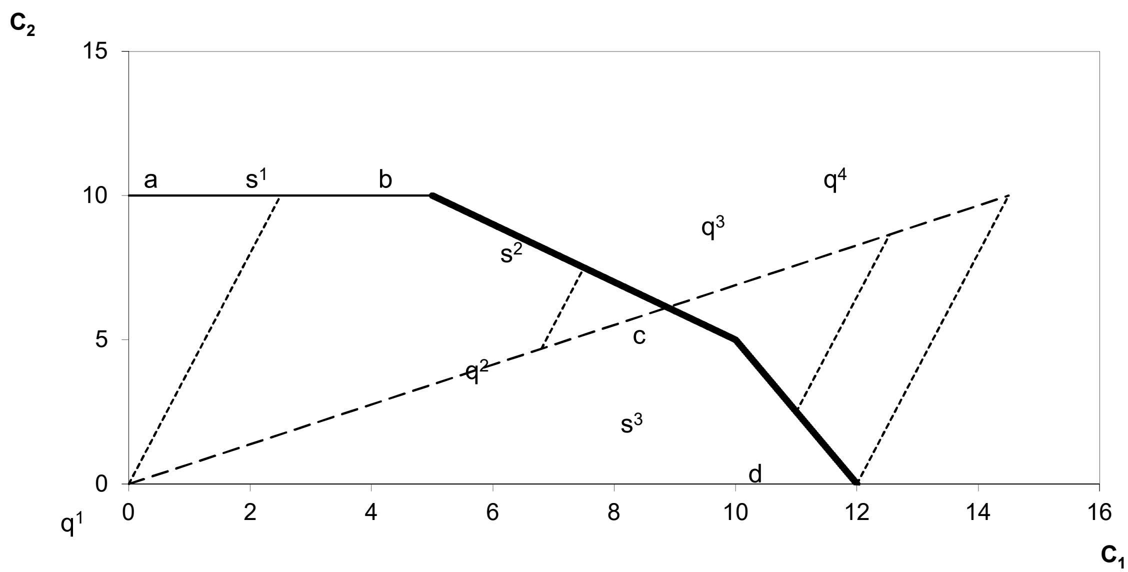
Andrzej Wierzbicki

ASP



Any nondominated point
can be reached!!

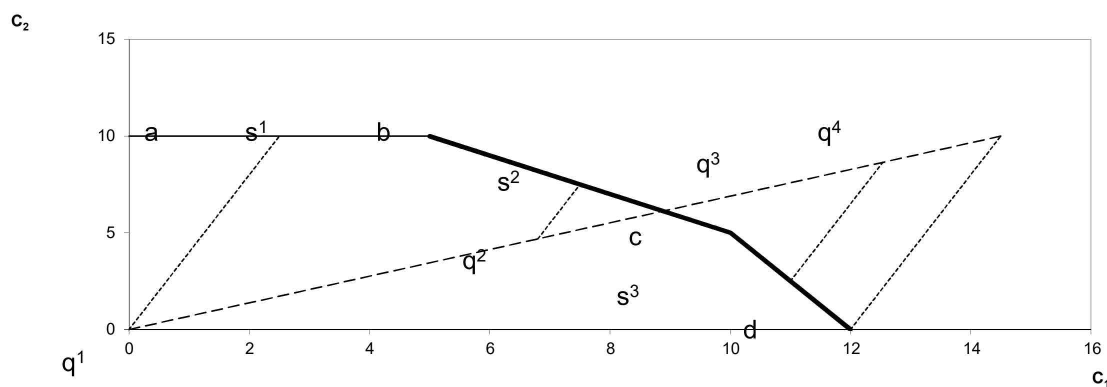
Example: λ_1=.8, λ_2=.2



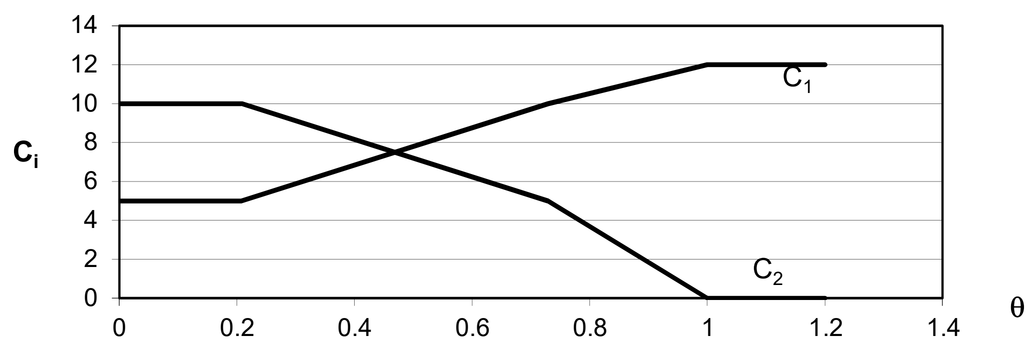
a. Projecting Solutions onto the Efficient Frontier

Parametric ASP (Korhonen and Laakso *EJOR* '86)

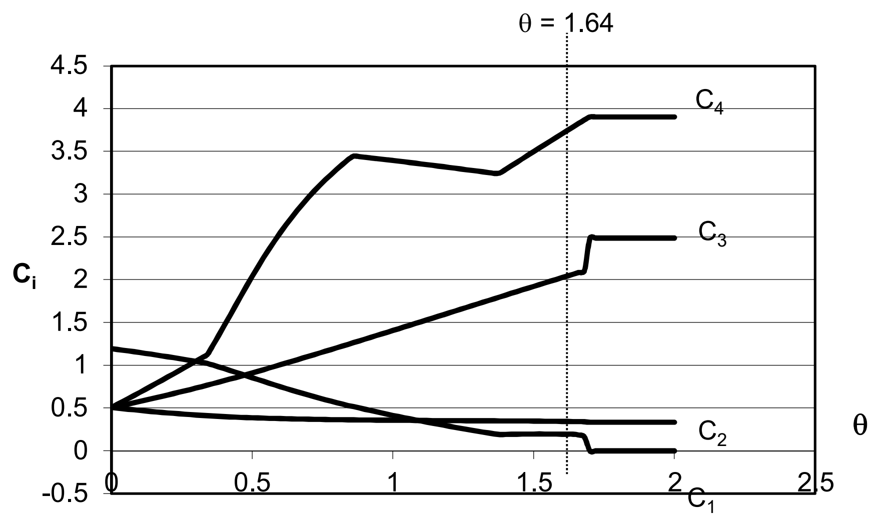
$$\begin{aligned} &Minimize \;\; \alpha - \varepsilon \sum_{i=1}^{m} z_i(x) \\ &s.t. \;\; \alpha \geq \lambda_i \left(q_i + \theta_i d_i - z_i(x) \right) \forall i \\ &\qquad x \in X \end{aligned}$$



a. Projecting Solutions onto the Efficient Frontier



b. Trajectory of Efficient solutions

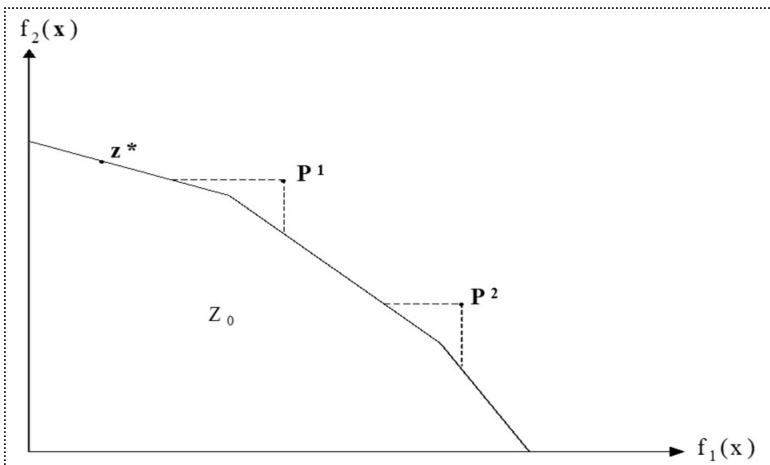


a. Step size vs. C_i values

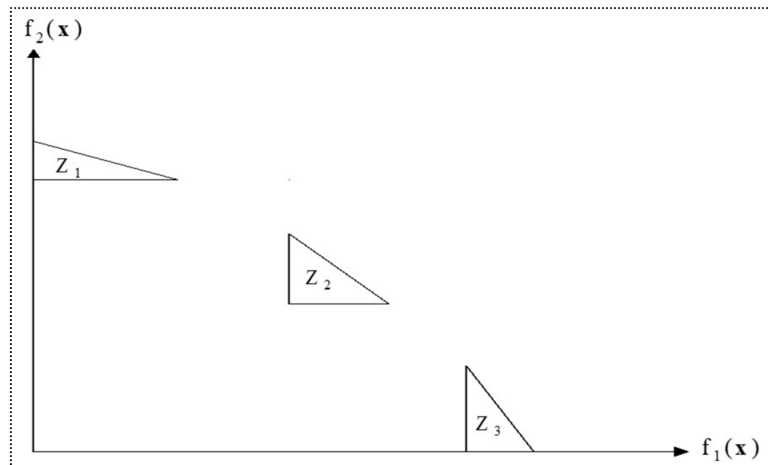
An Iteration for a Nonlinear Product Design Problem

General Monotone v (Köksalan and Karasakal *JORS* '06)

Assume z* is preferred to P^1 and P^2

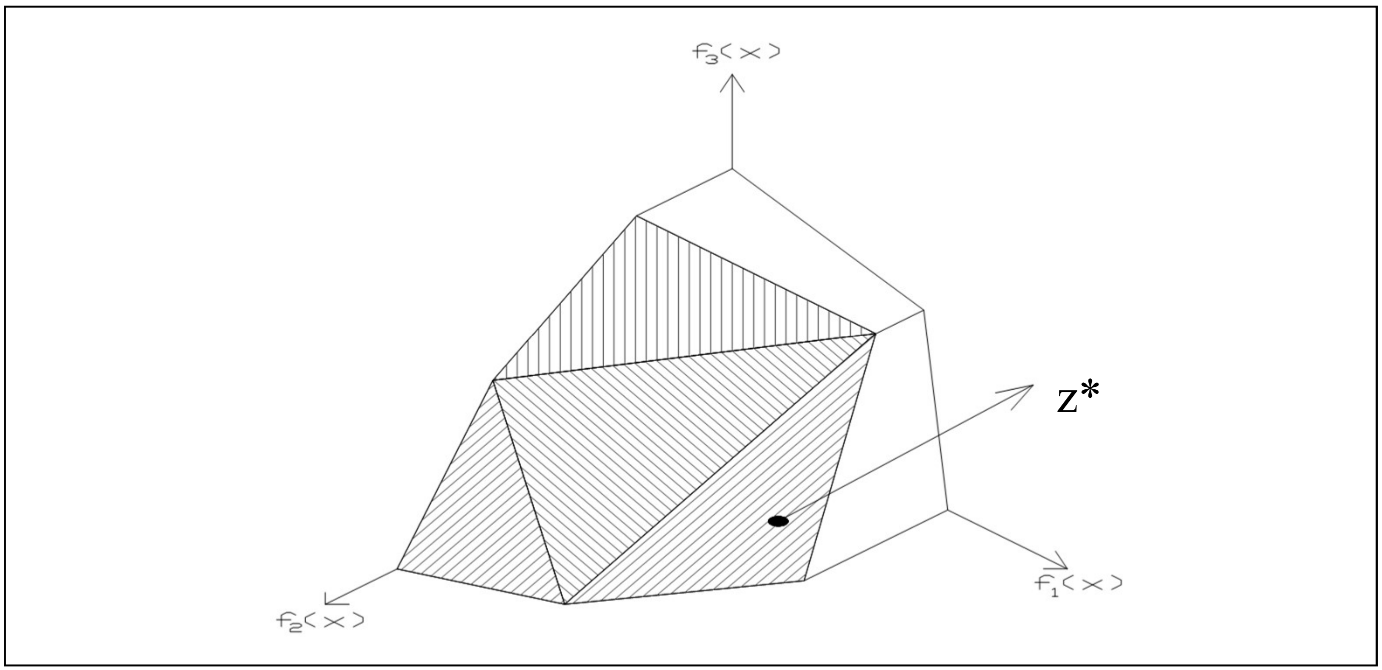


Original space

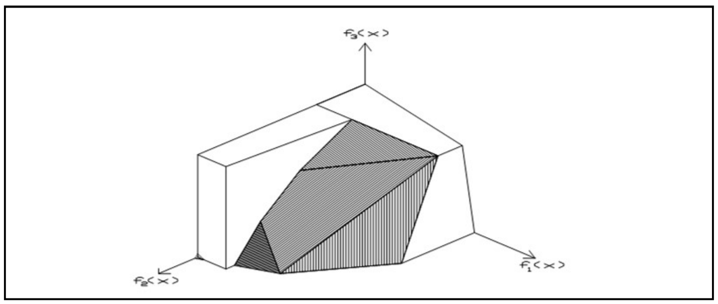


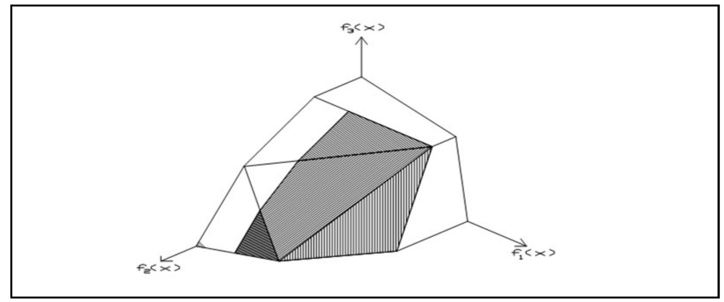
Reduced space

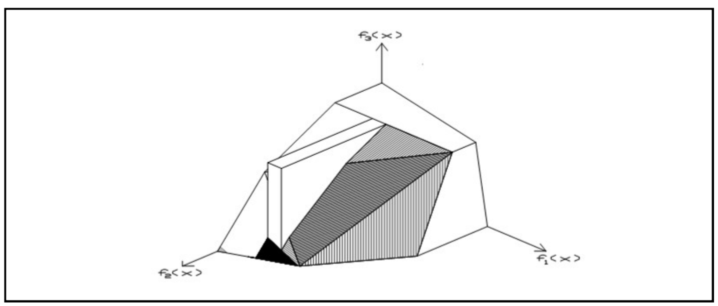
Example

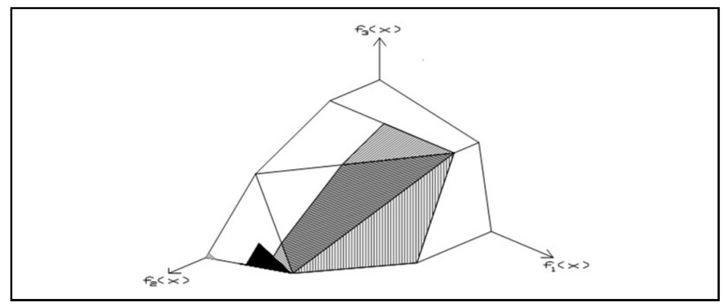


Example (cont.)

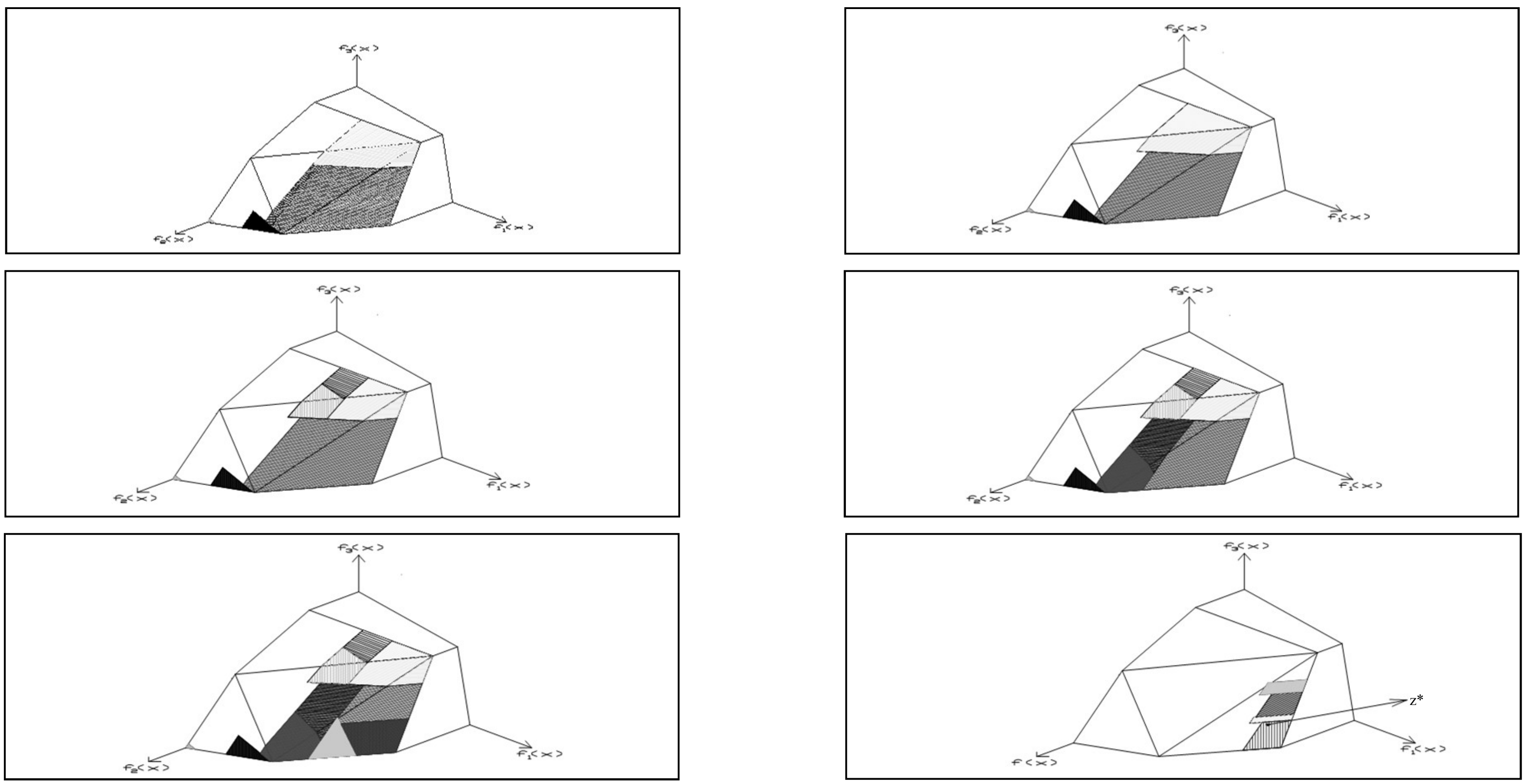








Example (cont.)



Nautilus-
Miettinen, Eskelinen, Ruiz, Luque, *EJOR* 2010

- Starts from the nadir point
- Gets direction information
- Works through dominated solutions to eventually reach the Pareto front

All “min” type

K. Miettinen et al./European Journal of Operational Research 206 (2010) 426–434

	Pref.	Visualization	min	low	current	max	Description
(a)		f1	-6.34	-6.34	-4.07	-4.07	Initial situation
		f2	-3.44	-3.44	-2.83	-2.83	
		f3	-7.50	-7.50	-0.32	-0.32	
		f4	-0.00	-0.00	9.71	9.71	
(b)	2,2,1,1	f1	-6.34	-6.33	-4.81	-4.07	Iteration 1
		f2	-3.44	-3.42	-3.00	-2.83	
		f3	-7.50	-7.49	-1.32	-0.32	
		f4	-0.00	0.65	8.35	9.71	
(c)	2,2,1,1	f1	-6.34	-6.31	-5.54	-4.07	It. 2. Backward step
		f2	-3.44	-3.39	-3.17	-2.83	
		f3	-7.50	-7.15	-2.32	-0.32	
		f4	-0.00	2.28	7.00	9.71	
(d)	2,3,1,4	f1	-6.34	-6.32	-5.56	-4.07	Iteration 3
		f2	-3.44	-3.35	-3.12	-2.83	
		f3	-7.50	-7.14	-1.79	-0.32	
		f4	-0.00	1.68	5.82	9.71	
(e)	1,2,1,2	f1	-6.34	-6.30	-6.30	-4.07	It. 4. Final solution
		f2	-3.44	-3.26	-3.26	-2.83	
		f3	-7.50	-2.60	-2.60	-0.32	
		f4	-0.00	3.63	3.63	9.71	

Fig. 3. Some iterations with NAUTILUS.

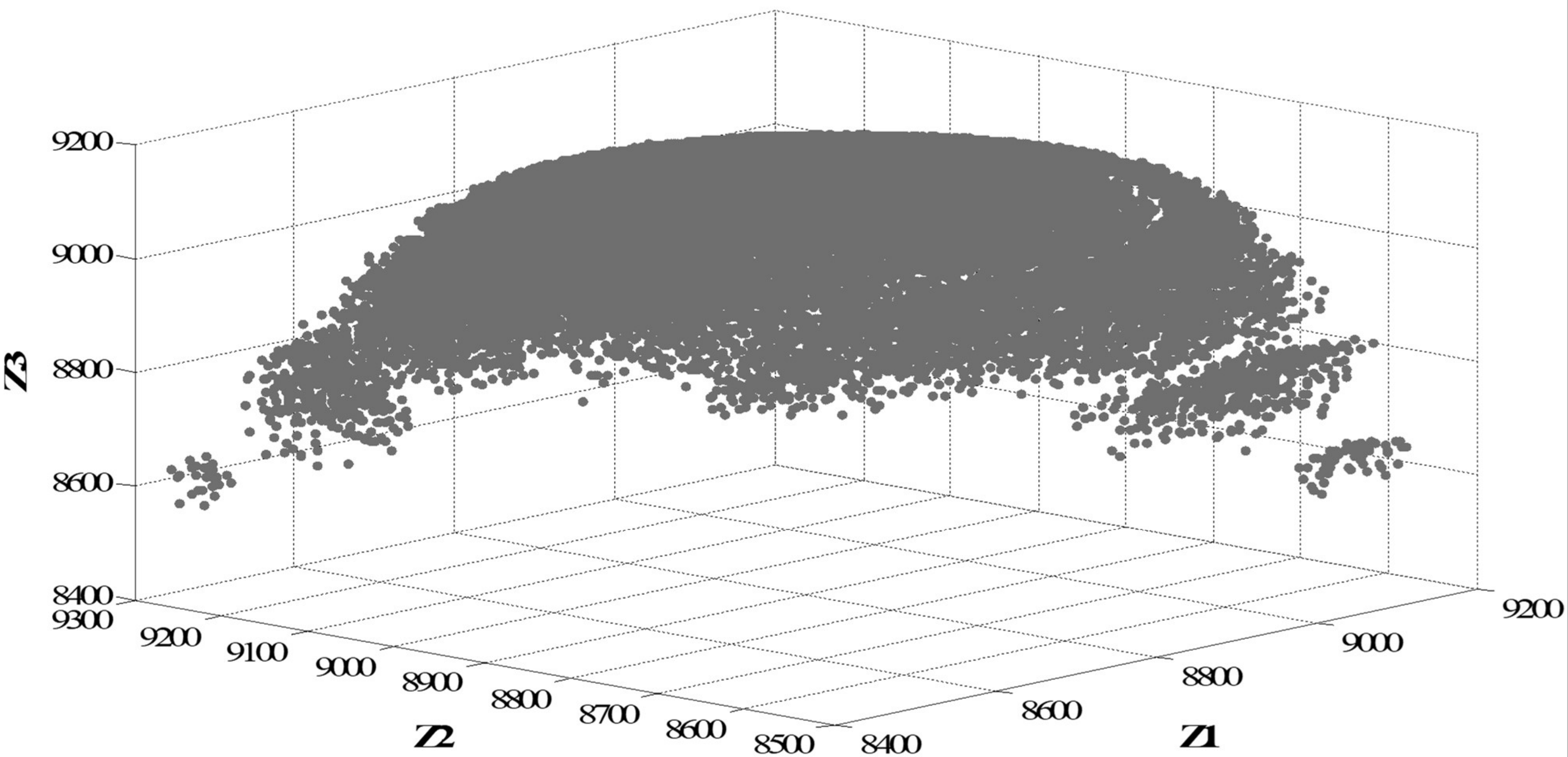
Combinatorial problems (MOCO)

- Mostly bicriteria approaches
- Many “optimize” a given v
- Some generate the nondominated set
- Many use modern heuristic search
- Few interactive approaches
- Review article (Ehrgott & Gandibleux ’00 *OR Spektrum*)
- Multi-objective Optimization using Evolutionary Algorithms (Deb ’01)
- Computationally hard

  (NP-complete, #P-complete)

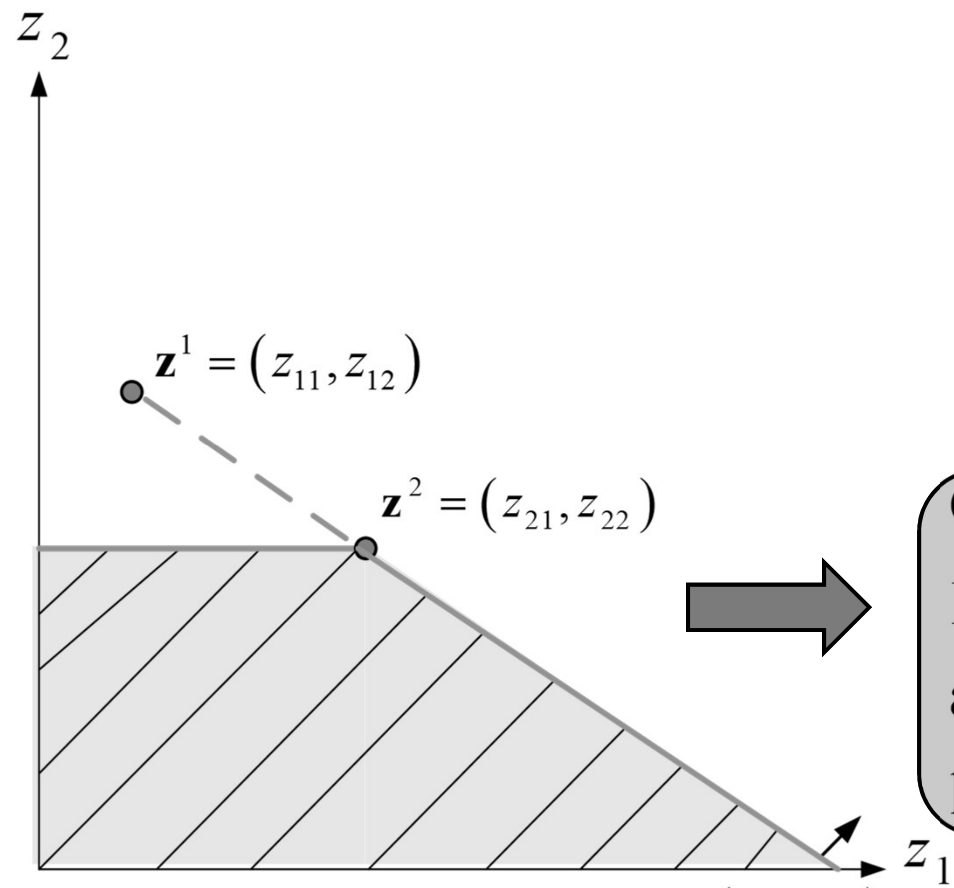
INTRODUCTION

200-item MOKP with 27260 nondominated points



QUASICONCAVE VALUE FUNCTION AND CONES
Lokman, Köksalan, Korhonen, Wallenius (*ANOR* 2016)

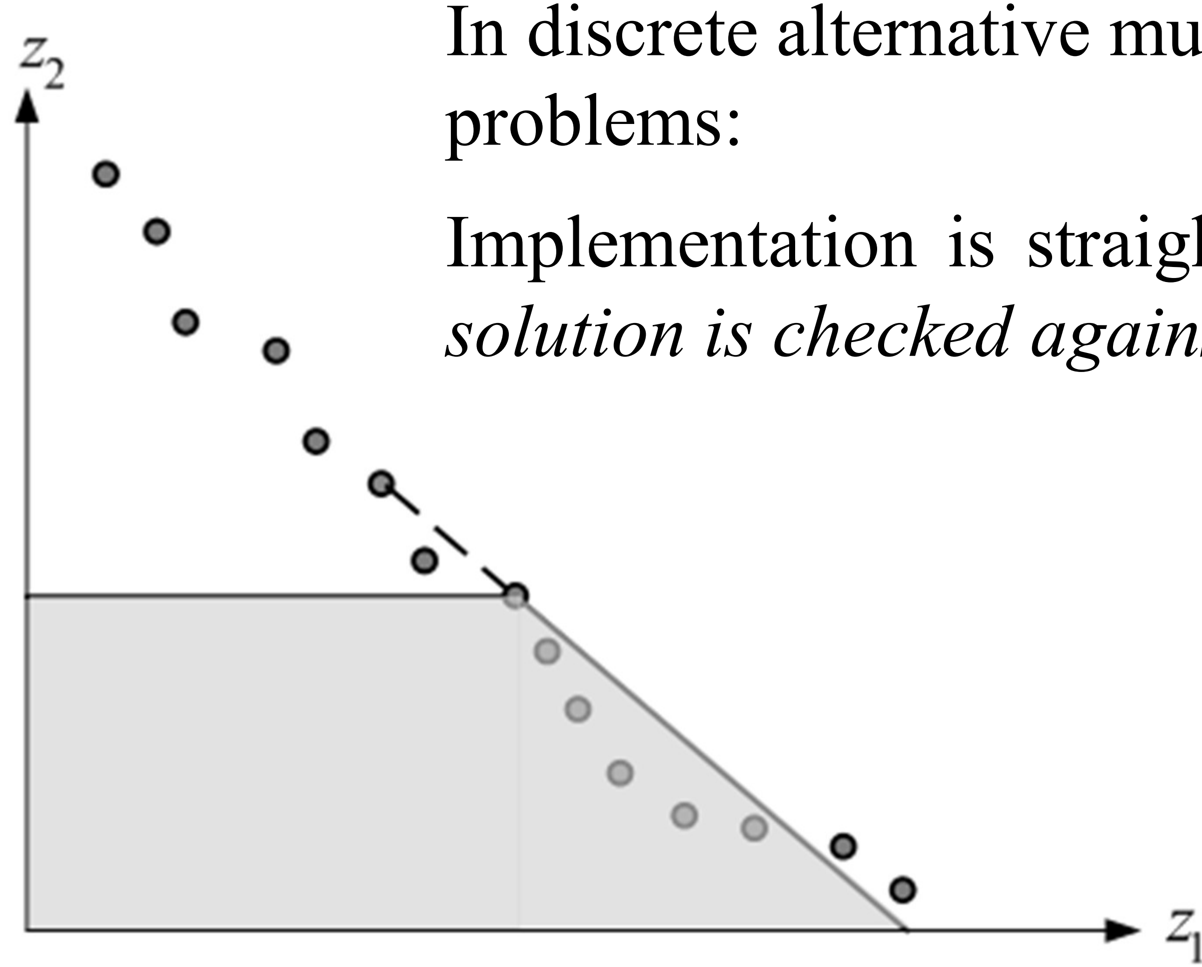
Demonstration of a 2-point convex cone



Cone dominated region
If $\mathbf{z}^1$ is preferred to $\mathbf{z}^2$,
all points in this region will be
less preferred than $\mathbf{z}^1$.

The region dominated by $C(\mathbf{z}^1; \mathbf{z}^2)$ – (Korhonen et al. 1984)

QUASICONCAVE VALUE FUNCTION AND CONES



In discrete alternative multi-objective problems:

Implementation is straight forward: *each solution is checked against each cone.*

QUASICONCAVE VALUE FUNCTION AND CONES

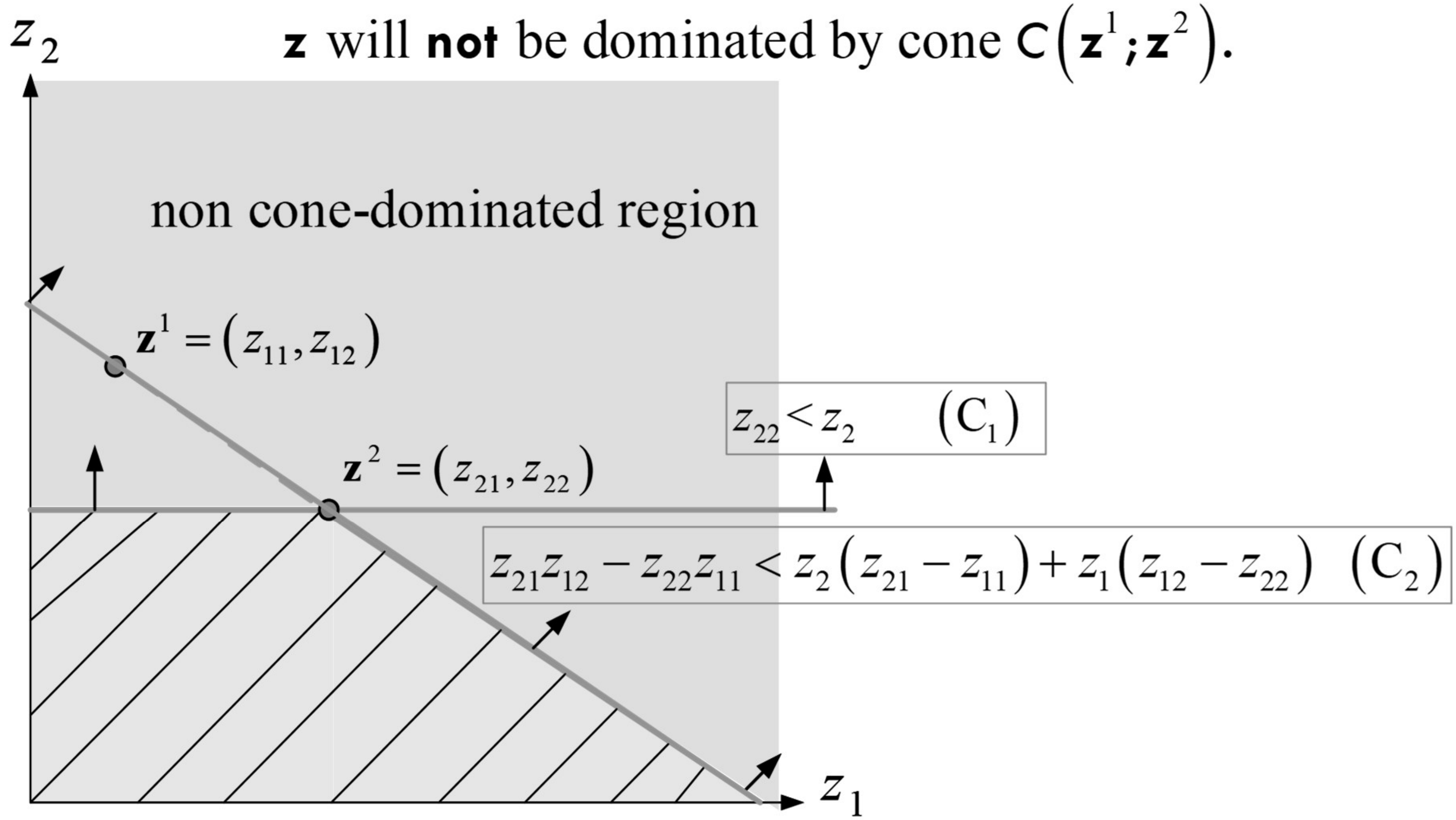
In our case:

- The solution space is defined by a set of constraints.
- Nondominated points are implicit.
- We need to characterize the admissible solution space: the non cone-dominated region.
- This region is typically nonconvex.
- Representing the admissible region is more manageable with 2-point cones.

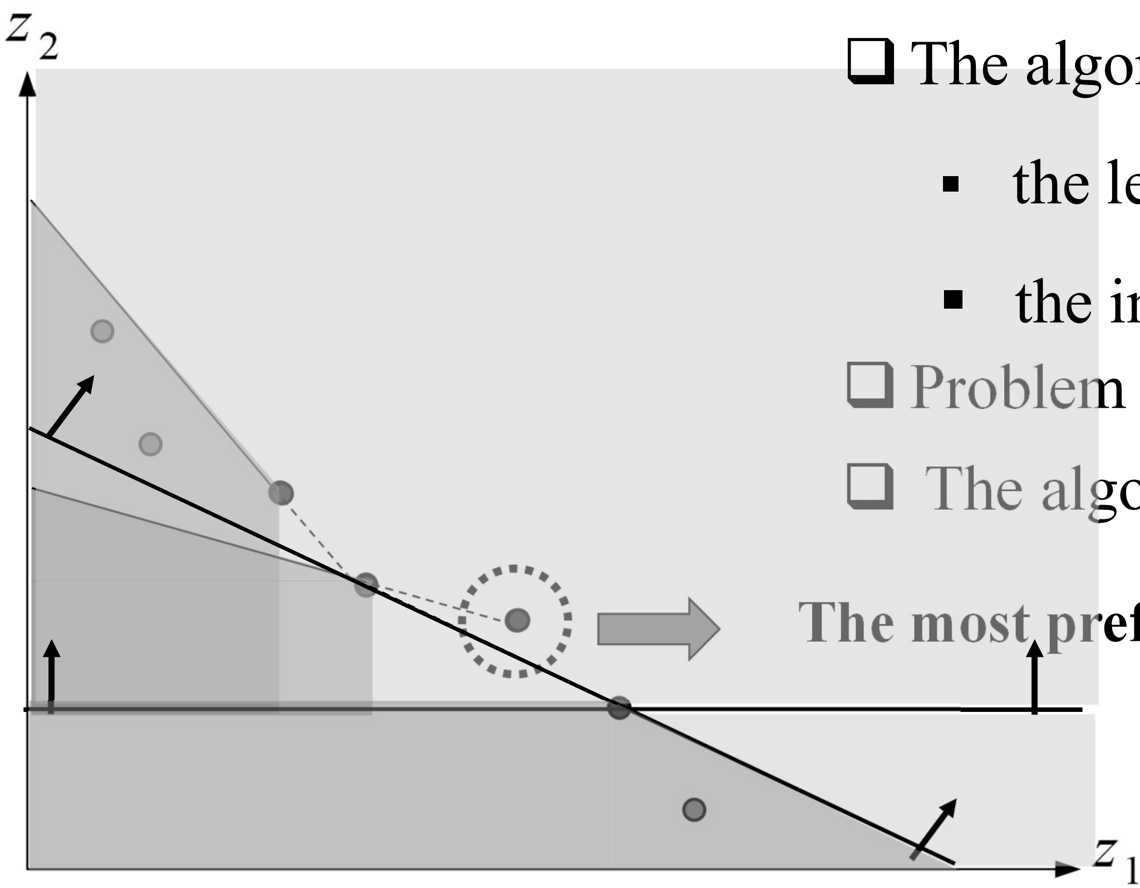
QUASICONCAVE VALUE FUNCTION AND CONES

Idea If $(\mathrm{C1})$ or $(\mathrm{C2})$ is satisfied, then we guarantee

$\mathbf{z}$ will **not** be dominated by cone $C\left(\mathbf{z}^1;\mathbf{z}^2\right)$.



AN INTERACTIVE ALGORITHM



- The algorithm excludes:
  - the less preferred regions
  - the incumbent point
- Problem becomes infeasible.
- The algorithm stops.

AN INTERACTIVE ALGORITHM

Solving MIP Problems Using Convex Preference Cones

Assuming quasiconcavity of DM’s value function:

- iteratively generates new nondominated points
- constructs 2-point convex cones based on the preferences of the DM.
- keeps an incumbent point and excludes inferior regions and the incumbent point.
- terminates when problem becomes infeasible since it implies all remaining nondominated points are inferior to the incumbent.
- guarantees finding the most preferred point

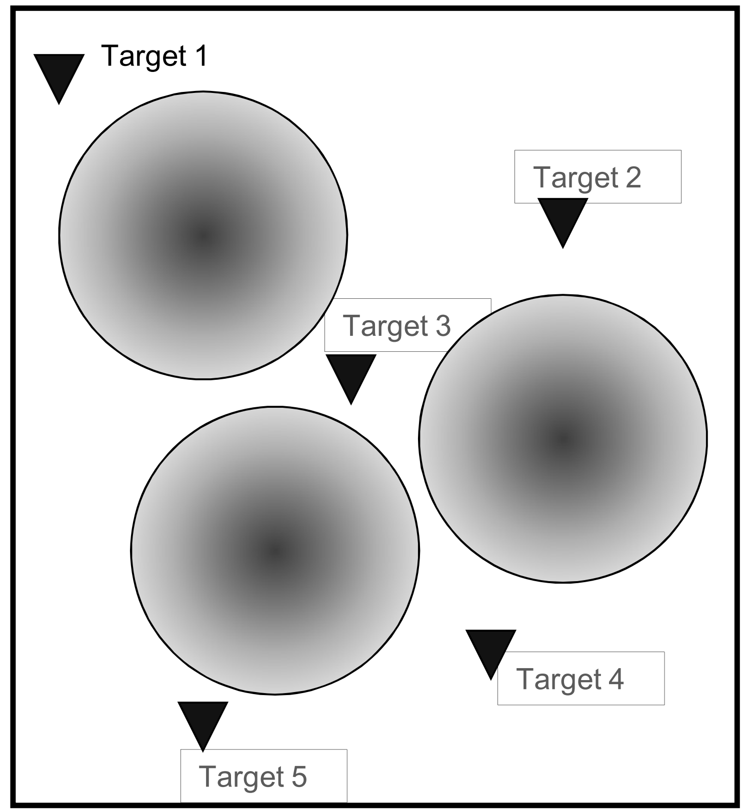
Diclehan Tezcaner Öztürk
Hacettepe University

UAV ROUTE PLANNING

Öztürk Tezcaner D. and M. Köksalan, *ANOR* (2016)

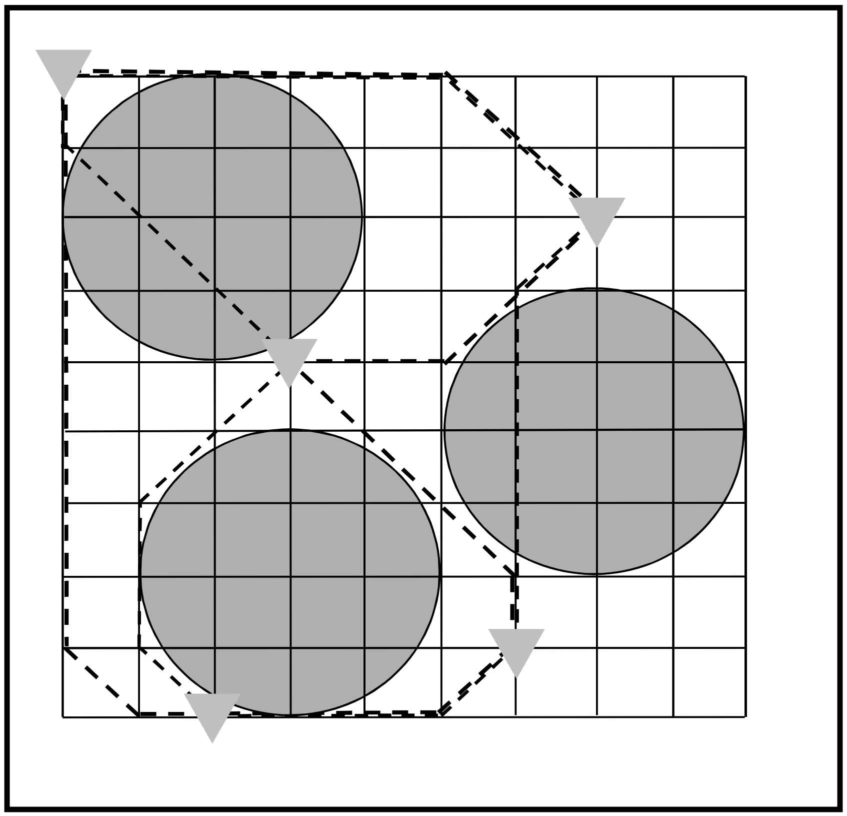
- UAV starts from a base, visits all targets, and returns to the base
- Minimize;
  - distance
  - radar detection threat

→ Biobjective Routing Problem

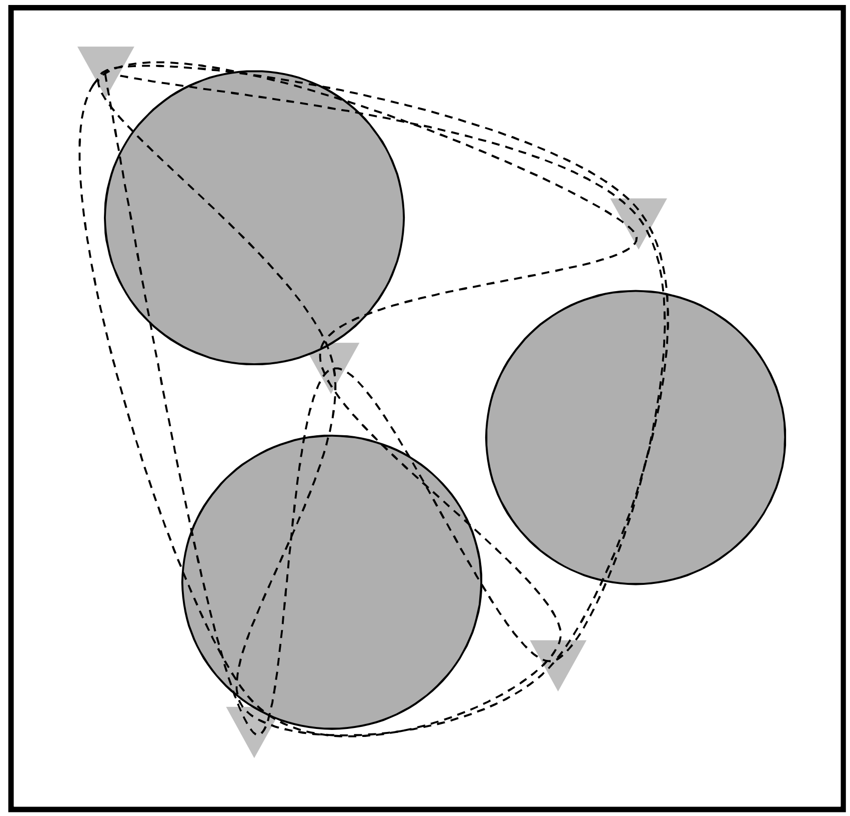


Terrain Types

Discretized Terrain



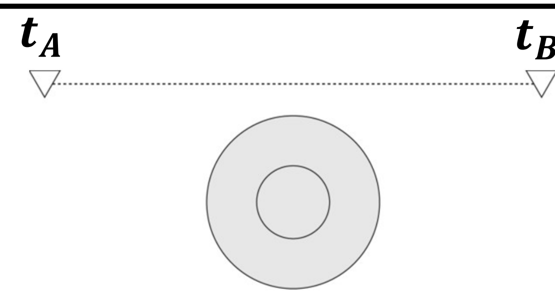
Continuous Terrain



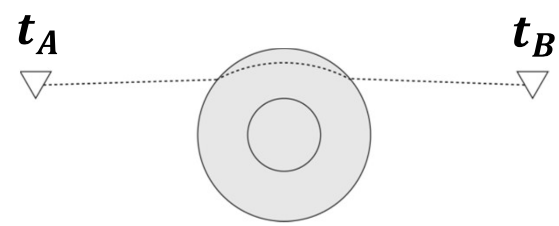
Movement Between Targets

Three types of moves:

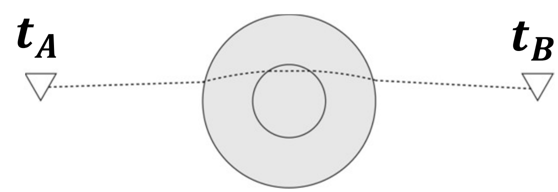
Type 1: No intersection with the radar region



Type 2: Moves through outer radar region only

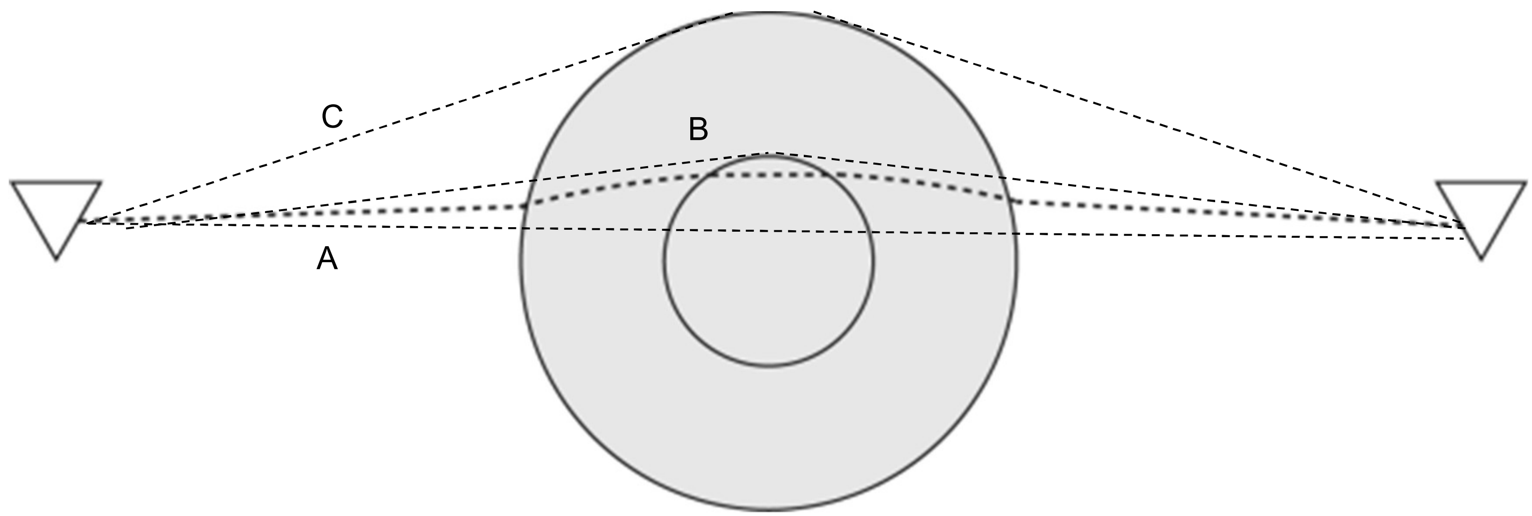


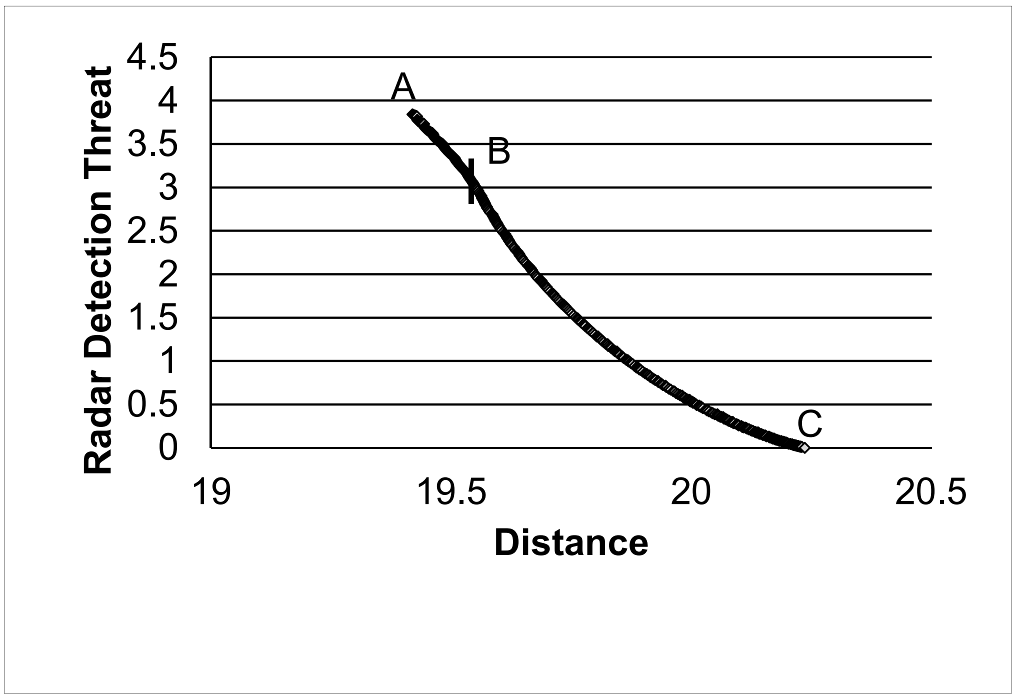
Type 3: Moves through both radar regions



$$D = \int_{t_A}^{t_B} ds$$

$$RDT = \int_{t_A}^{t_B} p_d . ds$$

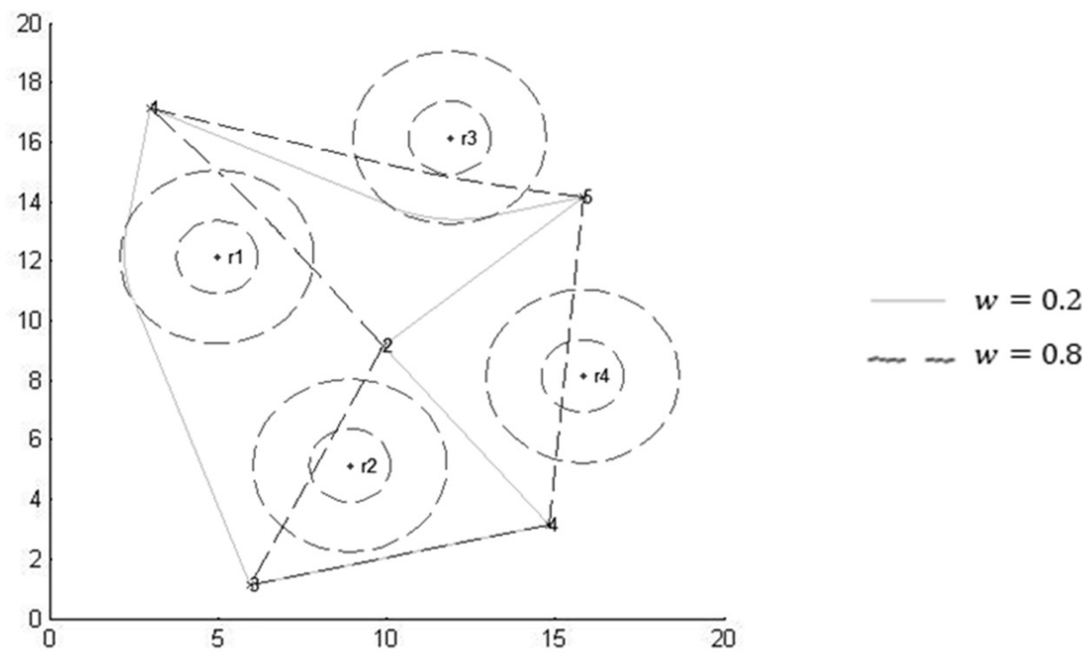

C
B
A


Radar Detection Threat
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
19
19.5
20
20.5
Distance
A
B
C

UAV Routing - Continuous Terrain

- Finding the most preferred solution of a DM with linear preference function

$U(z) = w.z_1(x) + (1 - w).z_2(x)$ where $0 < w < 1$



Routes found for $w = 0.2$ and $w = 0.8$

Recent Software

- iMOLPe - interactive Multi-Objective Linear Programming explorer
- Visualization of results obtained with TRIMAP, STEM, ICW and Pareto Race interactive methods.
- Free download: http://www.uc.pt/en/org/inescc/software

Some References

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Resources

International Society on MCDM:

http://www.mcdmsociety.org/ : Free membership thru Web site

International Conferences

- 25th International Conference, Summer 2019, Istanbul, Turkey, Ilker Topcu.
- 24th International Conference, July 9-14, 2017, Ottawa, Canada, Sarah Ben Amor.
- 23rd International Conference, August 3-7, 2015, Hamburg, Germany, Martin J. Geiger.
- 22nd International Conference, June 17-21, 2013, Málaga, Spain, Francisco Ruiz.
- 21st International Conference, June 13-17, 2011, Jyväskylä, Finland, Kaisa Miettinen
- 20th International Conference, June 22-26, 2009, Chengdu, China, Yong Shi, S. Wang
- 19th International Conference, January 7-12, 2008, Auckland, New Zealand, M. Ehrgott
- 18th International Conference, June 19-23, 2006, Chania, Greece, C. Zapounidis
- 17th International Conference, August 6-11, 2004, Vancouver, Canada, Bill Wedley
- 16th International Conference, 2002, Semmering, Austria, M. Luptacik, R. Vetschera
- 15th International Conference, July 10-14, 2000, Ankara, Turkey, Murat Köksalan

…

- 1st International Conference, 1975, Jouy-en-Josas, France, H. Thiriez, S. Zionts

Resources (cont.)

Other groups:

- **INFORMS Section on MCDM**
- **EURO Working Group on MCDA (MCDA ‘84, Vienna, Austria, September 22-24, 2016)**
- **GP**
- **MOEA**
- **…**

Publications

- Many books
- Journal of MCDA
- Springer Proceedings
- Regular OR Journals
- Other Specialized Journals

History of MCDM

Multiple Criteria Decision Making: From Early History to the 21st Century by M. Köksalan, J. Wallenius and S. Zionts, World Scientific, 2011.