Decision Modelling I

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Outline

Who is she?

Why we are not psychotherapists

language: formal (reducing ambiguity)
abstract (domain independent)

Models of rationality

Should we become?

- Can we control our profession?
- Do we have manuals for novices?
- Do we have protocols for assessment?
- Do we have specific training?

What are we looking for?

A Science?

- Decision Science?
- Decision Aiding Science?
- What is a Science?

We need a Decision Aiding Methodology

A coherent structure of reasoning about theories and practices concerning deciding and aiding to decide.

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Deciding ...

- Decision Maker
- Decision Process
- Cognitive Effort
- Responsibility
- Decision Theory

... and Aiding to Decide

- A client and an analyst
- Decision Aiding Process
- Cognitive Artifacts
- Arguing and Convincing
- Decision Aiding Methodology

What Decision Aiding means?

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Consensual construction of shared cognitive artifacts

Decision Aiding makes sense only with respect to a Decision Process in which the client is involved and with respect to which demands advice.

What is a Decision Aiding Process?

A Decision Aiding Process is a Decision Process where at least two actors are involved: the client and the analyst, with at least two concerns: the client's "problem" and the analyst's job, mobilising at least the following resources: the client's domain knowledge and the analyst's methodological knowledge.

A Decision Aiding Process becomes part of the Decision Process for which it has been established. The analyst enters as an actor such a Decision Process.

Its Cognitive Artifacts

- Representation of the problem situation
- Problem Formulation
- Evaluation Model
- Final Recommendation

Representing a Problem Situation

- Who has a problem?
- Why this is a problem?
- Who other is affected by the decision process?
- Who decides?
- Who pays for the consequences and the bill?
- What I am doing here?

A Problem Situation

$$PS = \langle \mathcal{A}, \mathcal{O}, RS \rangle$$

- $\mathcal{A}$ actors, participants, stakeholders
- $\mathcal{O}$ objects, concerns, stakes
- RS resources, commitment

Formulating a Problem

Constructing a first formal representation of the client's concerns, applying an abstract and formal language, using a model of rationality.

- What objects do we consider in formulating "the problem"?
- What do we know or are we looking for such objects?
- What do we want to do with such objects?

A Problem Formulation

$$\Gamma = \langle \mathbb{A}, \mathbb{V}, \Pi \rangle$$

- $\mathbb{A}$ Actions
- $\mathbb{V}$ Points of view
- Π Problem statement

Constructing an Evaluation Model

- Fixing alternatives.
- How to describe them?
- Are there any preferences?
- How to put all this information together?

Evaluation Model

$$\mathcal{M} = \langle A, (D, E), H, \mathcal{R} \rangle$$

- A alternatives, decision variables, ...
- D dimensions, attributes, ...
- E scales associated to attributes,
- H values, scenarios, opinions, preference models, ...
- $\mathcal{R}$ procedures, algorithms, protocols ...

Establishing a final Recommendation

- Going back to reality.
- What do we put in the final report?
- Is it valid?
- Is it legitimated?
- It works?
- Are we satisfied?

Meaningfulness ...

- Do we use the information correctly?
- Is it meaningful for the analyst?
  (Measurement Theory)
- Does it make sense for the decision process?
- Is it meaningful for the client?
  (Client Satisfaction)

... and Legitimation

- Ownership
- Organisational Dimension
- Culture
- Decision Process

Which Rationality Principles?

Being rational while deciding, or being rational while aiding to decide?

Substantive Rationality

To respect goals to achieve.

Procedural Rationality

To respect procedures to follow.

Bounded Rationality

To feel satisfied by the results at hand.

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But this does not fit ...

Is the analyst convinced about what to do and if not how to convince her/him?
Is the client convinced about what to do and if not how to convince her/him?
Will the result be accepted within the decision process and if not how to make it acceptable?

But this does not fit ...

Is the analyst convinced about what to do and if not how to convince her/him?
Is the client convinced about what to do and if not how to convince her/him?
Will the result be accepted within the decision process and if not how to make it acceptable?

Practically:

Decision Aiding is about constructing convincing arguments (for the client, the analyst and the decision process) about what to do.

Conceptions of rationality

"Rationality"

refers to the quality of acts and decisions which is increased by agents when they thematise and modify aspects of their acts and decisions so as to improve their ability to counter criticisms.

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- strategic: objective independent truths;
- norm regulated: legitimate behaviours and expectations;
- dramaturgic: importance of subjective satisfaction;
- communicative: all the above.

Decision aiding approaches

- Objectivist: focussing on strategic conceptions of rationality.
- Conformist: focussing on morm-regulated conceptions of rationality.
- Adjustive: focussing on dramaturgic conceptions of rationality.
- Reflexive: focussing on communicative conceptions of rationality.

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what appears to be binding in the problem situation? What arguments and counter arguments do we expect?

- Unquestionable formulations and solutions.
- Unquestionable decision aiding architecture.
- Unquestionable spirit of initiative.
- Nothing unquestionable.

Problems

- Patients triage in emergency room;
- Identification of classes of similar DNA sequences;
- Star ratings of hotels;
- Waste collection vehicle routing;
- Vendor rating and bids assessment;
- Optimal mix of sausages;
- Chip-set lay out;
- Airplanes priority landing;
- Tennis tournament scheduling ...

What is important?

Characterisation

Is there a way to characterise problems which does not depend from the methods used?

What does really matter?

In designing, choosing, applying, implementing, understanding, explaining, justifying, a method?

What are the primitives?

And what is the derived information and the expected outcomes?

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What is my perspective?

A general theory for scholars and practitioners such that:

- it is easy to teach;
- it is easy to use “practically”;
- it is easy to implement (if necessary ...).

What is a decision problem?

Consider a set A established as any among the following:

- an enumeration of objects;
- a set of combinations of binary variables;
- a set of profiles within a multi-attribute space;
- a vector space in $\mathbb{R}^n$.

Without loss of generality:

$A \subseteq \prod_j X_j$ where $\forall j X_j = \{x_{1j}, \cdots x_{nj}\}$, X_j being separable and ordered;

Technically:

A Decision Problem is a partitioning of A under some desired properties.

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Problem Statements

Practically we partition A in n classes. These can be:

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Two special cases:
- there are only two classes (thus complementary);
- the size (cardinality) of the classes is also predefined.

Claims 1

Recall: $A \subseteq \prod_j X_j$

A decision problem exists iff
- $\exists X_j$ such that X_j is known and
- $\forall X_j$ such that X_j is unknown these are not separable.

Claim 1

Establishing the set A is on its turn a decision problem. We get a fix point A_0 using the known attributes and we establish a recursion where each step of is a decision problem.

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All the previously mentioned problems boil down in aggregating some ordering relations applied on the set A.

Claim 3

From an algorithmic point of view a decision problem boils down to an optimisation algorithm.

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What is a ranking problem?

Primitive

The input is a binary relation on A: $\succeq \subseteq A \times A$ to be read "at least as good as".

Result

The result is a partitioning of A in $[A_1], \cdots [A_n]$ such that:
$[A_j] \geq [A_i] \Leftrightarrow j \geq i$ and
$\forall x \in [A_j], y \in [A_i] : x \succeq' y$

Discussion 1

What is a choice problem?

We partition A in two classes $[A_1] \geq [A_2]$. Thus $[A_1] = \sup_A(\succeq')$.

What is an optimisation problem?

A choice problem for which:

- $\succeq = \succeq'$
- $x \succeq y \Leftrightarrow f(x) \geq f(y)$.
- Thus $[A_1] = \max_A f(x)$

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Discussion 2

Why is $\succeq'$ different from $\succeq$?

Generally speaking $\succeq$ is not an ordering relation since preferences can be partial and or inconsistent. If we have to proceed with some operational procedure we need to transform $\succeq$ to an ordering relation $\succeq'$.

Discussion 2

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How do we learn $\succeq$?

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How do we learn $\succeq$?
What properties should $\succeq'$ fulfill?

What is a clustering problem?

Primitive

The input is a set of binary relations on A: $\approx_l \subseteq A \times A$
to be read "similar to".

Result

The result is a partitioning of A in $[A_1], \cdots [A_n]$ such that:
$\exists \approx_l : \forall x, y \in [A_j] \;\; x \approx y$ and
$\forall x \in [A_j], \;\; y \in [A_i] : \neg(x \approx y)$

Discussion 1

Indiscernibility.

In case $\approx_I$ are equivalence relations then the partitioning of A results in constructing the indiscernibility relation on A. However, this is not generally the case and $[A_j] = \sup_A(\approx_I)$.

In other terms we try to maximise similarity within classes (clusters) and minimise similarity among classes (clusters).

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Discussion 2

Distances.

If $\approx_l$ are nested similarity relations with nice properties then we can establish a metric:
- $s(x, y)$: how similar is x to y?
- $d(x, y)$: how distant is x from y?

Then $[A_y] = \{x | \max_A F(s(x, y))\}$,
F being a measure of the overall similarity of the elements of $[A_y]$ with respect to y.

What properties should F and the metrics fulfill?

What is a rating problem?

Primitive

The input is a binary relation on A: $\succeq \subseteq A \times P \cup P \times A$
to be read "at least as good as".
P being the set of external "norms" characterising the ordered classes $C_1 \rhd \cdots \rhd C_n$

Result

The result is to assign each element of A in a C_j such that:
$x \in C_j \Leftrightarrow x \succeq' p_j, p_{j+1}, \cdots p_n$ and $p_1 \cdots p_{j-1} \succeq' x$

Discussion 1

Constraint Satisfaction

If $\forall x, y \in A \cup P \;\; x \succeq y \;\Leftrightarrow\; f(x) \geq f(y)$.
Then $x \in C_j \;\Leftrightarrow\; f(p_{j-1}) \geq f(x) \geq f(p_j)$.
This is a Constraint Satisfaction Problem.

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What is an assigning problem?

Primitive

The input is a set of binary relations on A: $\approx_I \subseteq A \times P \cup P \times A$ to be read "similar to".
P being the set of external "norms" characterising the classes $C_1 \cdots C_n$

Result

The result is to assign each element of A in a C_j such that:
$x \in C_j \Leftrightarrow \exists \approx_I : x \approx_I p_j$

Discussion 1

Constraint Satisfaction

If $\forall x, y \in A \cup P \;\; x \approx_I y \;\Leftrightarrow\; f(x) = f(y)$.
This is once again a Constraint Satisfaction Problem.

Why optimisation is important?

Let's call the ranking and clustering problems "unsupervised" decision problems and the rating and assignment ones "supervised" ones.

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- **Any supervised decision problem is a constraint satisfaction problem**.

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Let's call the ranking and clustering problems "unsupervised" decision problems and the rating and assignment ones "supervised" ones.

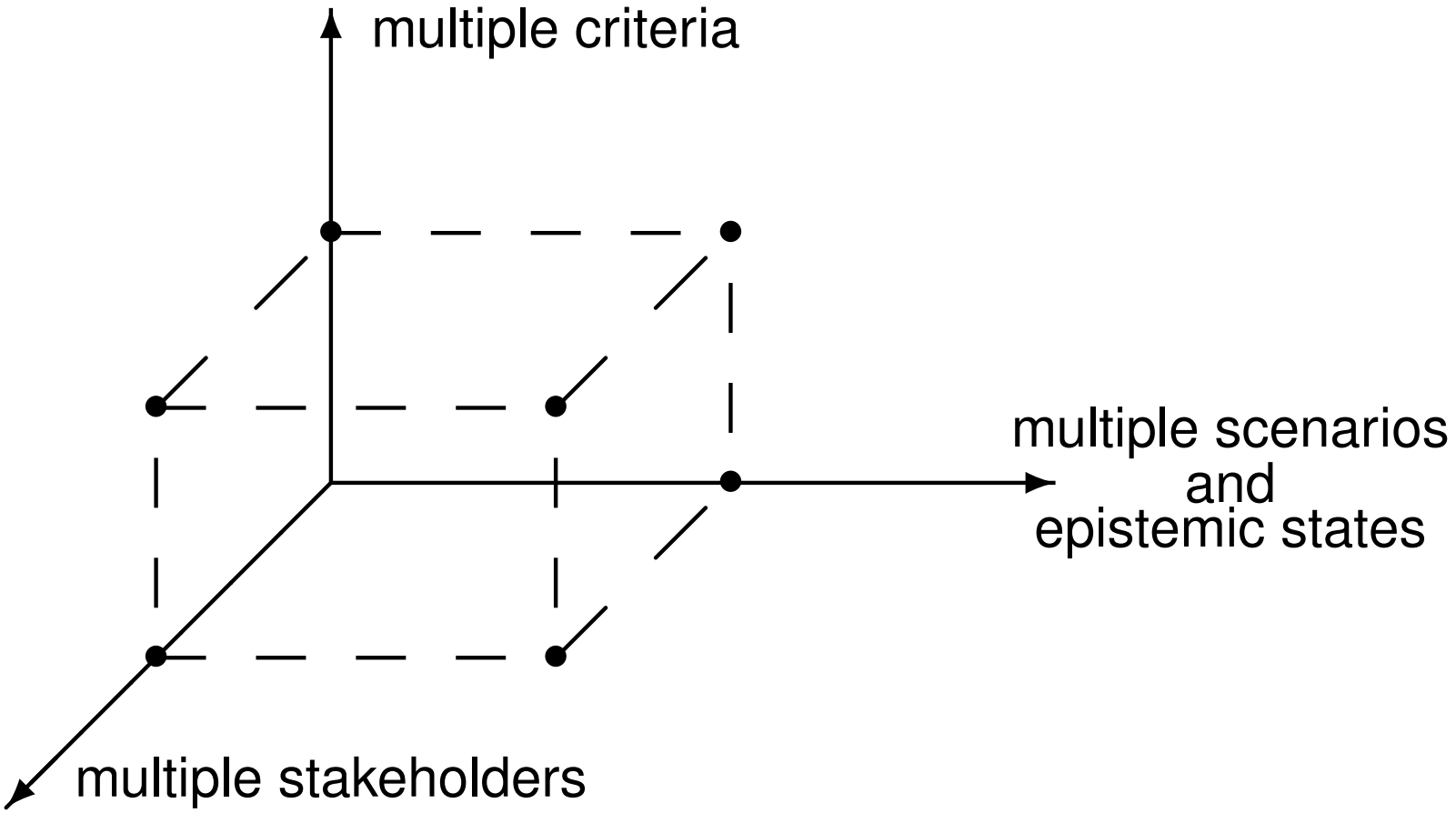
- **Any unsupervised decision problem is an optimisation problem**.
- **Any supervised decision problem is a constraint satisfaction problem**.

Since any constraint satisfaction problem can be seen as an optimisation problem, we can definitely focus only to the later ones

Why aiding to decide is not straightforward?

- multiple opinions
- multiple values
- multiple scenarios
- + algorithmic aspects

Where primitives come from?



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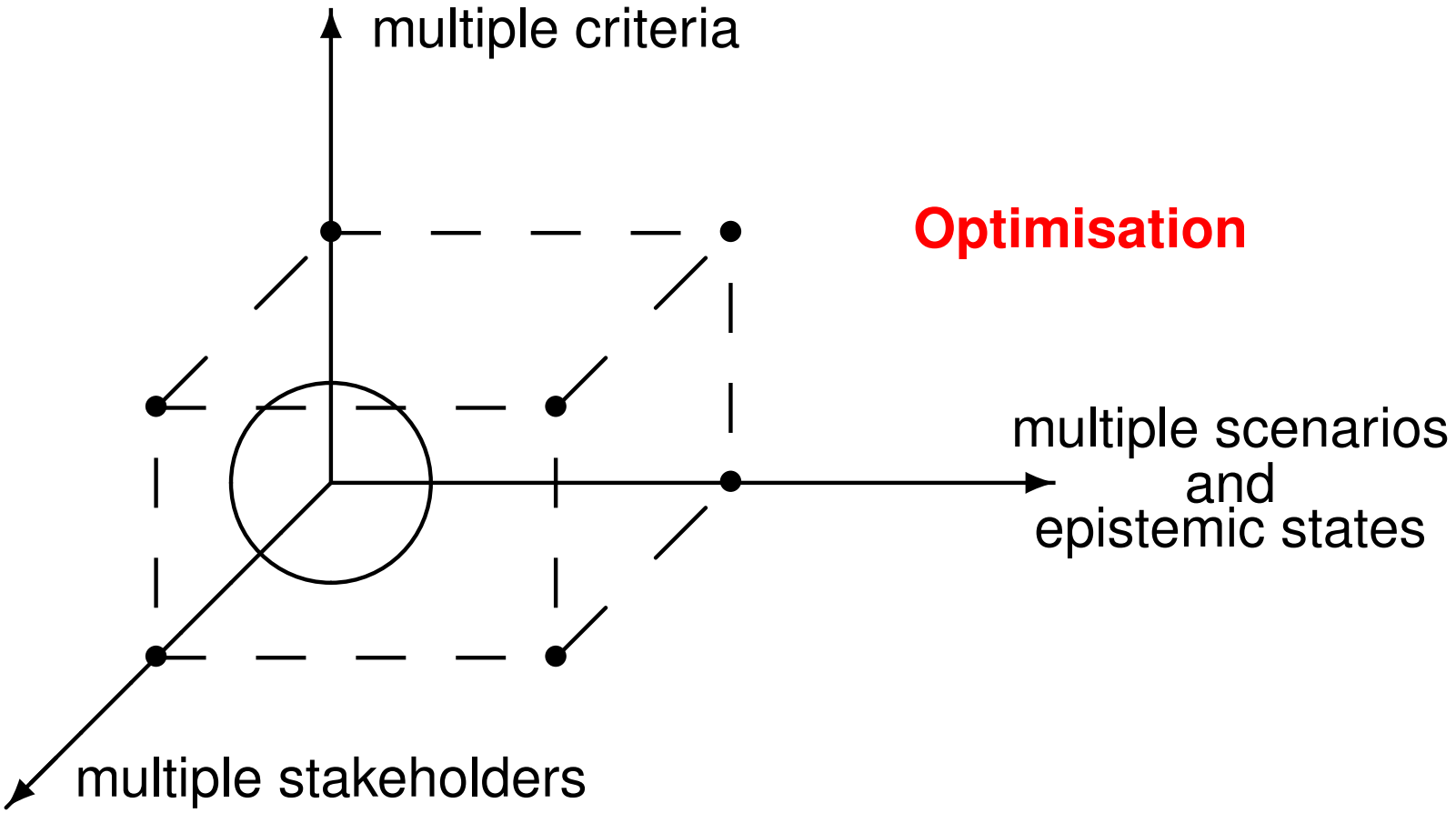

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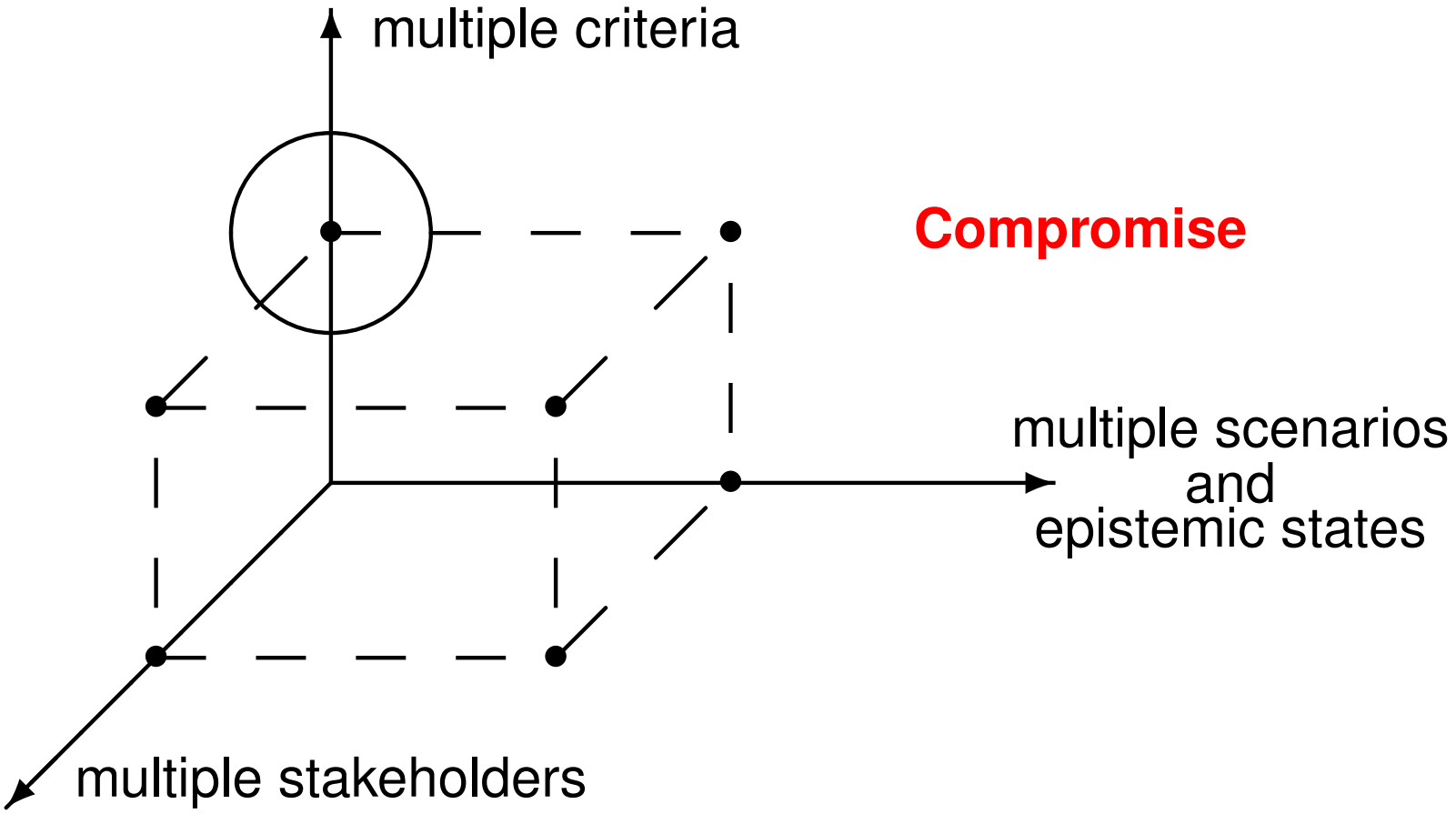

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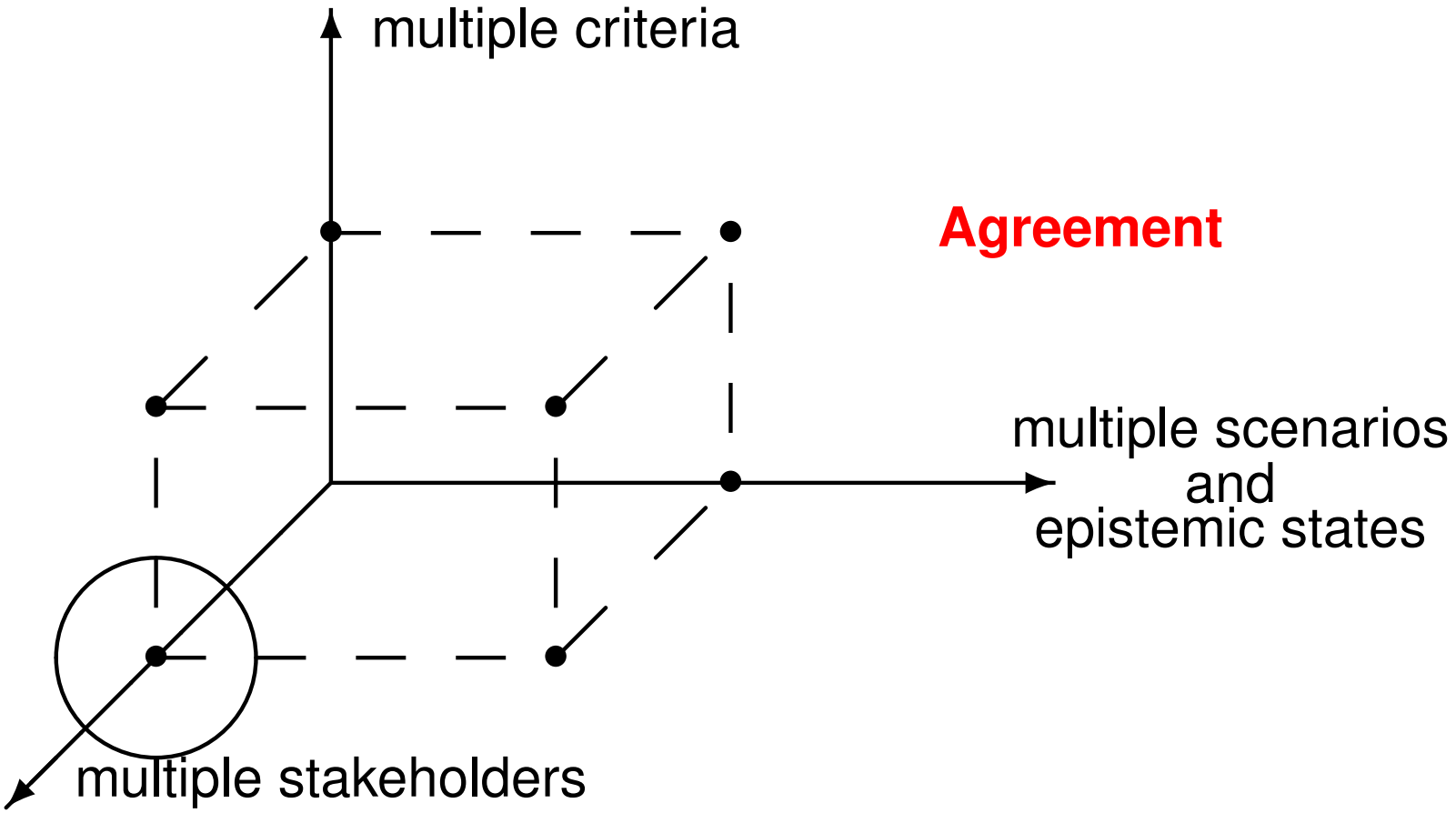

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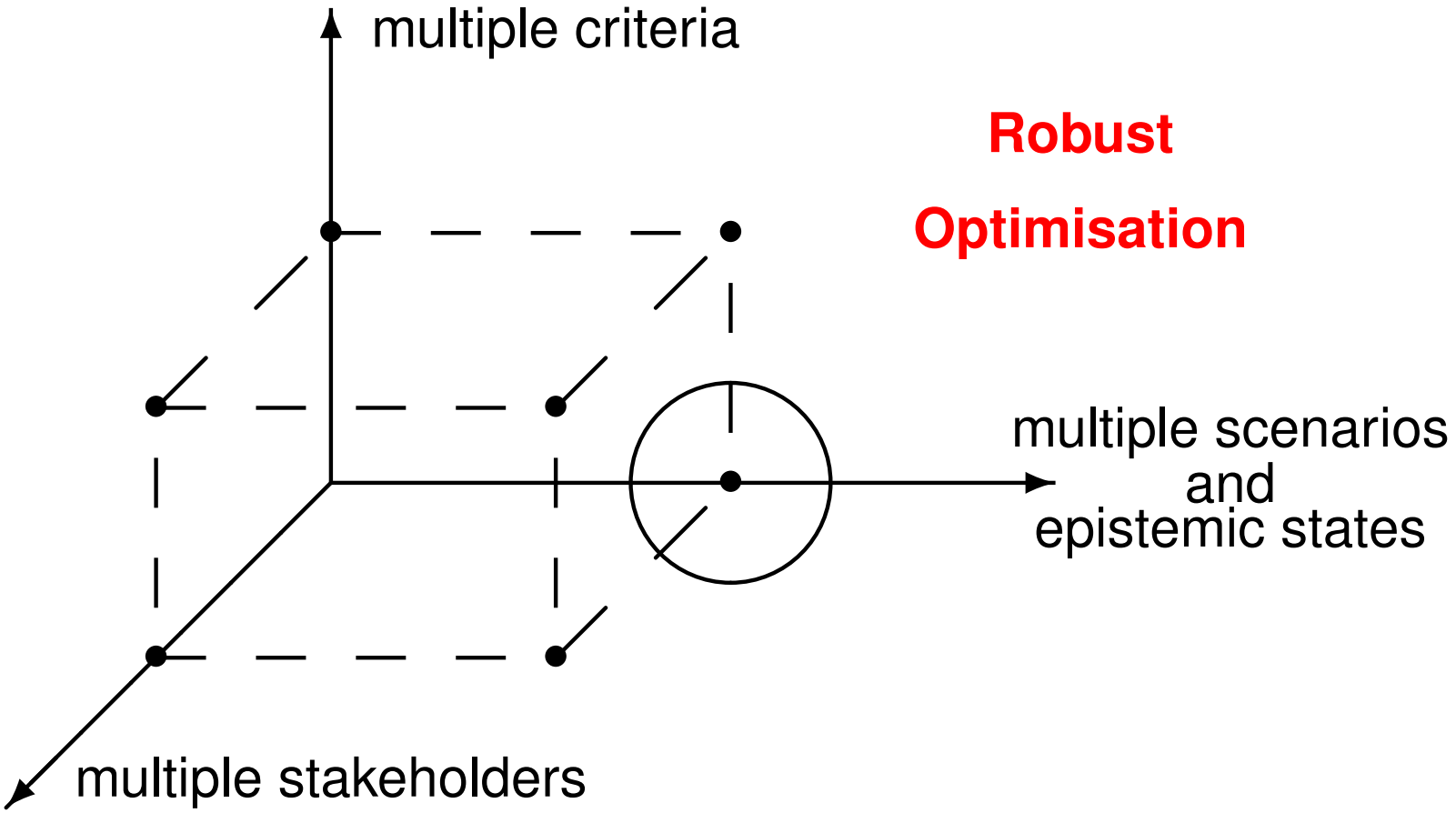

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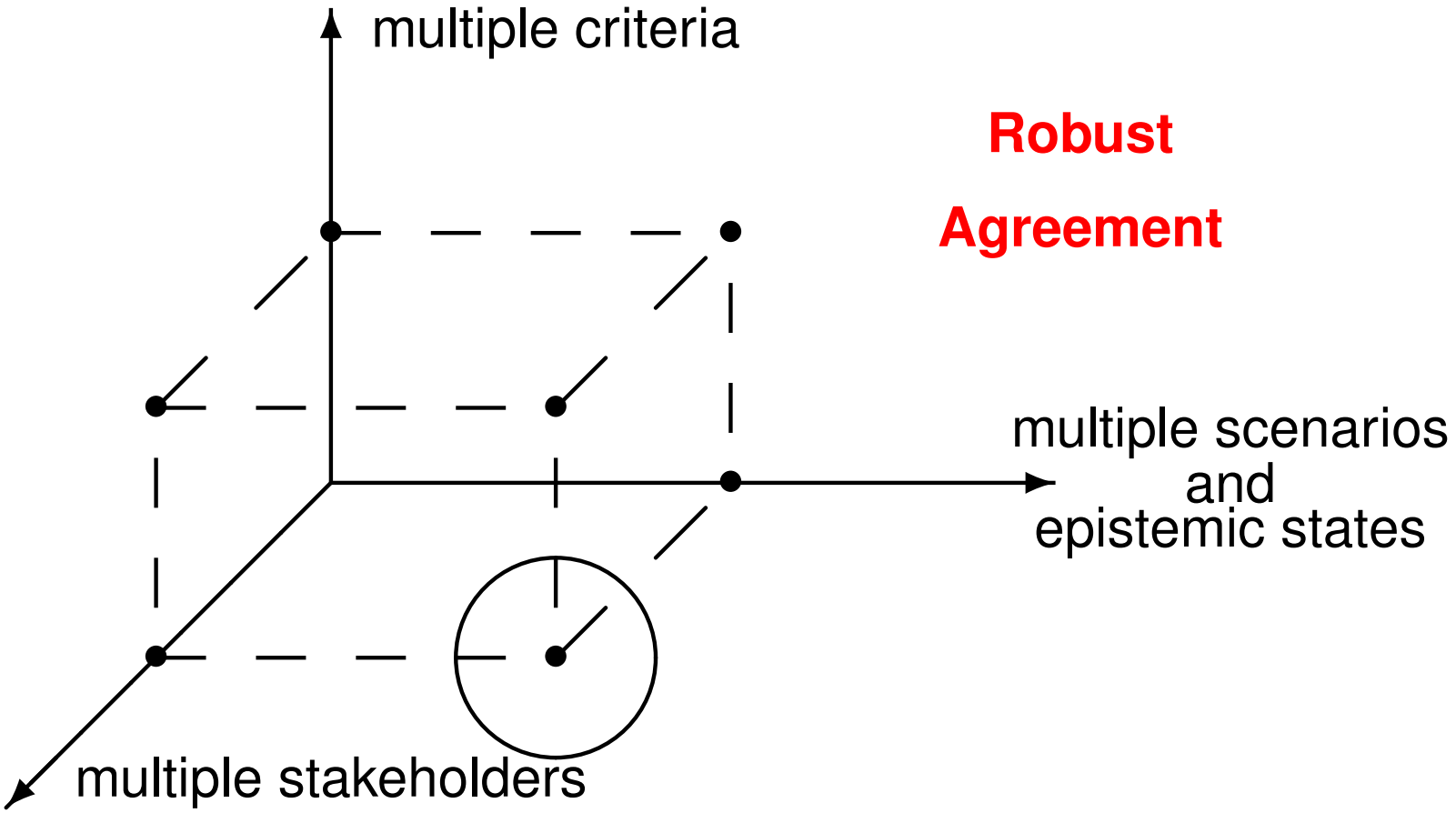

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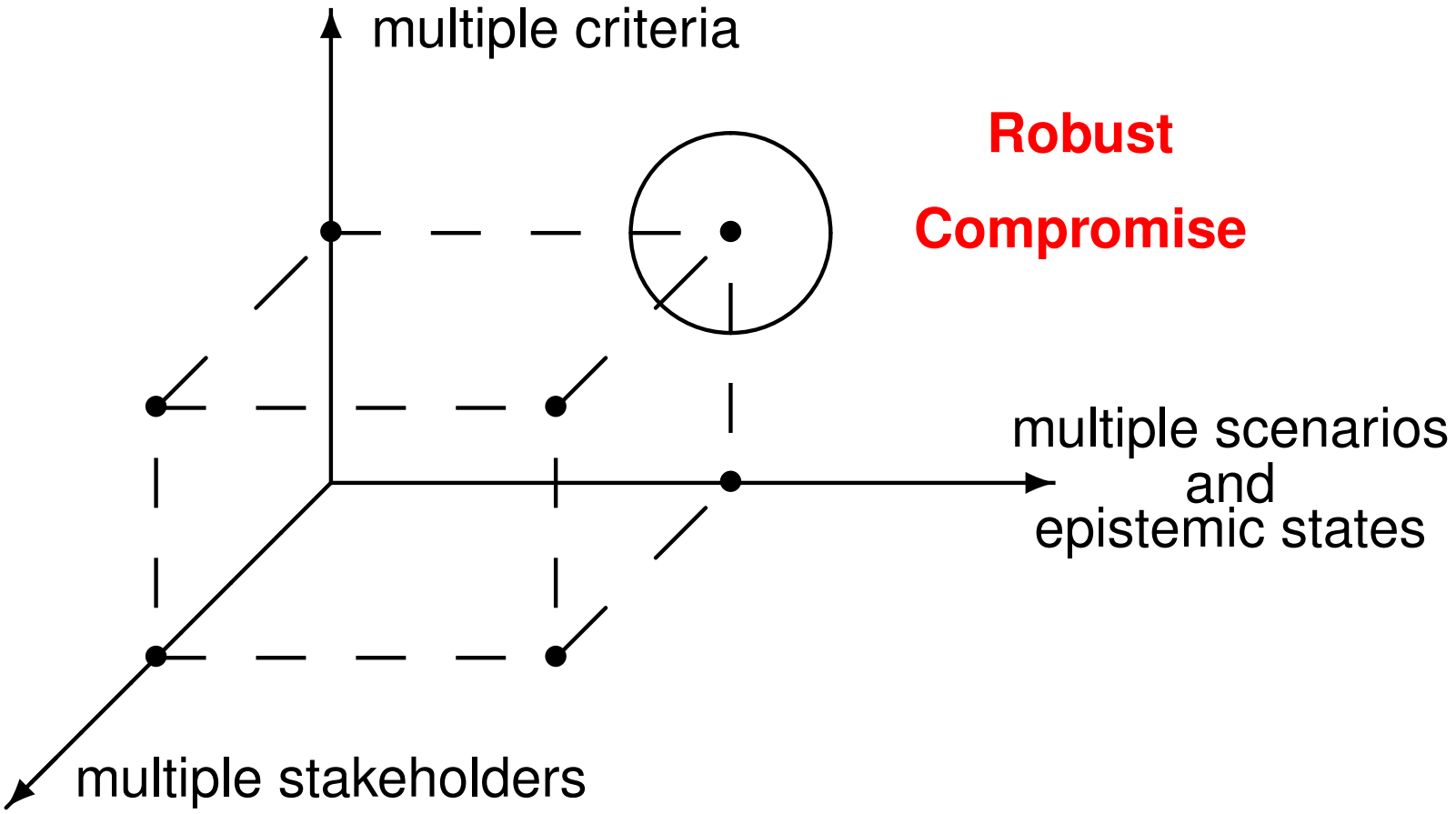

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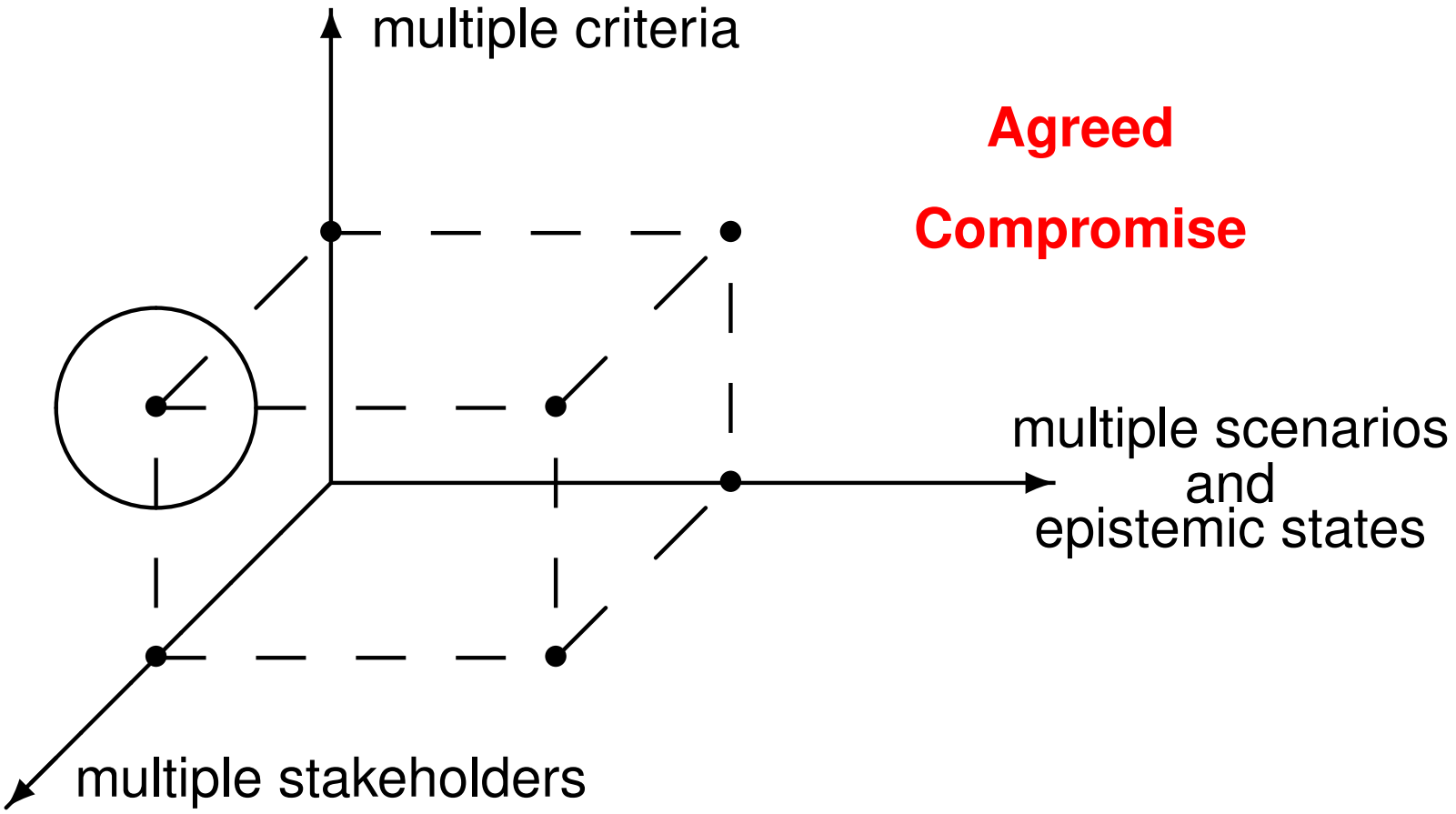

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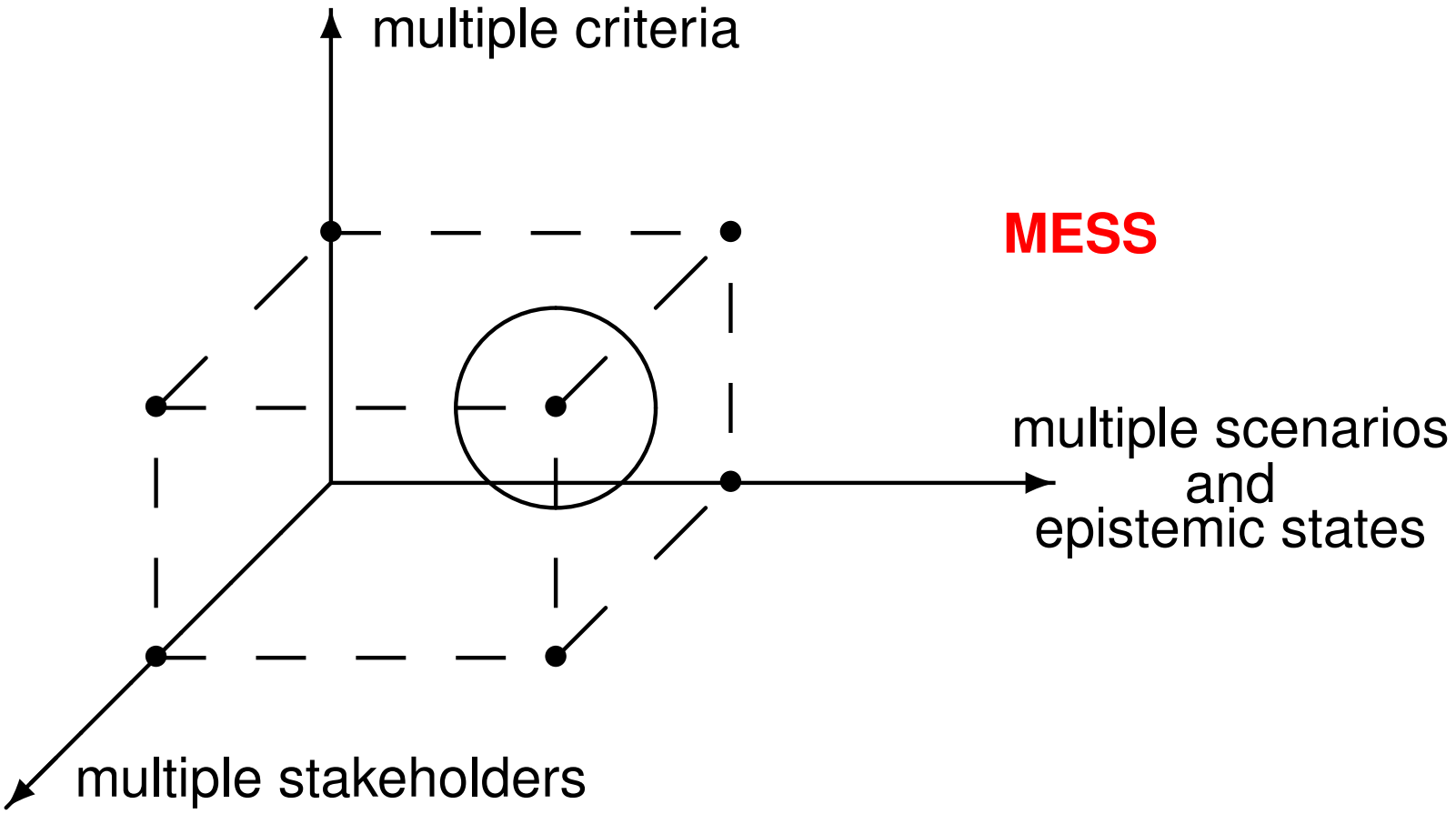
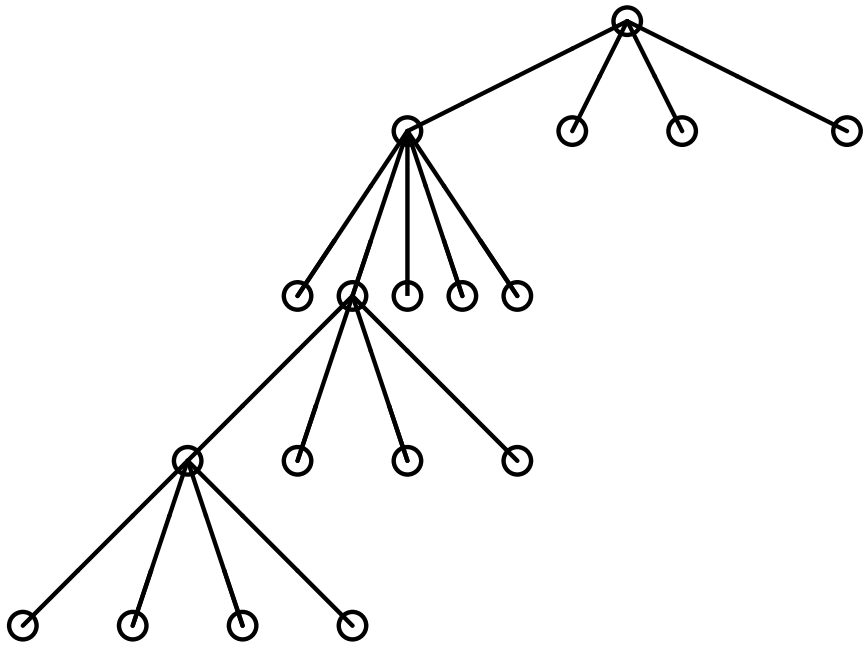


Is that all?

- Behind a criterion other criteria may be considered in a hierarchy of criteria (objectives);
- Behind a stakeholder other actors may have to be considered, that precise stakeholder being a speaker for a community;
- Behind a state of the nature other uncertainties may have to be considered;
- Any combination of the above may in reality occur as complex as possible.

Hierarchy

A decision problem can be represented as a sequence of preference aggregations along an hierarchy of actors, criteria and states of the nature, combined arbitrarily.



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- We need procedures/protocols allowing to learn such “preferences”.
- We need procedures/algorithms allowing to aggregate such “preferences” before moving to the next node of the hierarchy.
- We need an intelligent tracking procedure allowing to construct ultimate arguments as well as to update and revise the model.